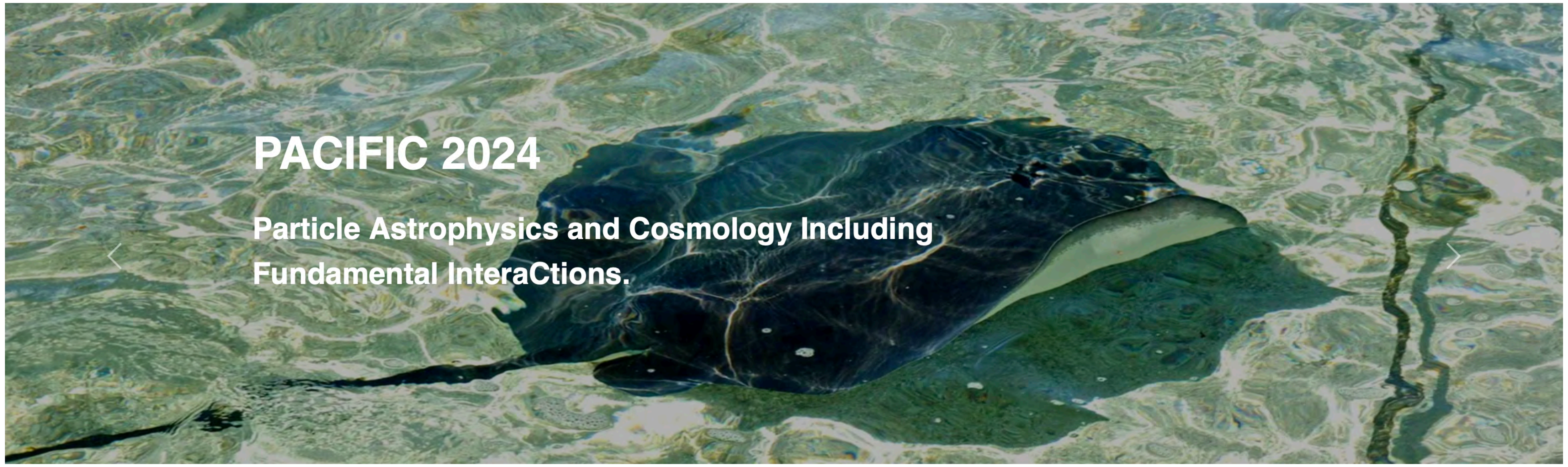
Spin-2 Scattering Amplitudes and Gravitational Portal Dark Matter

Elizabeth H. Simmons
R. Sekhar Chivukla

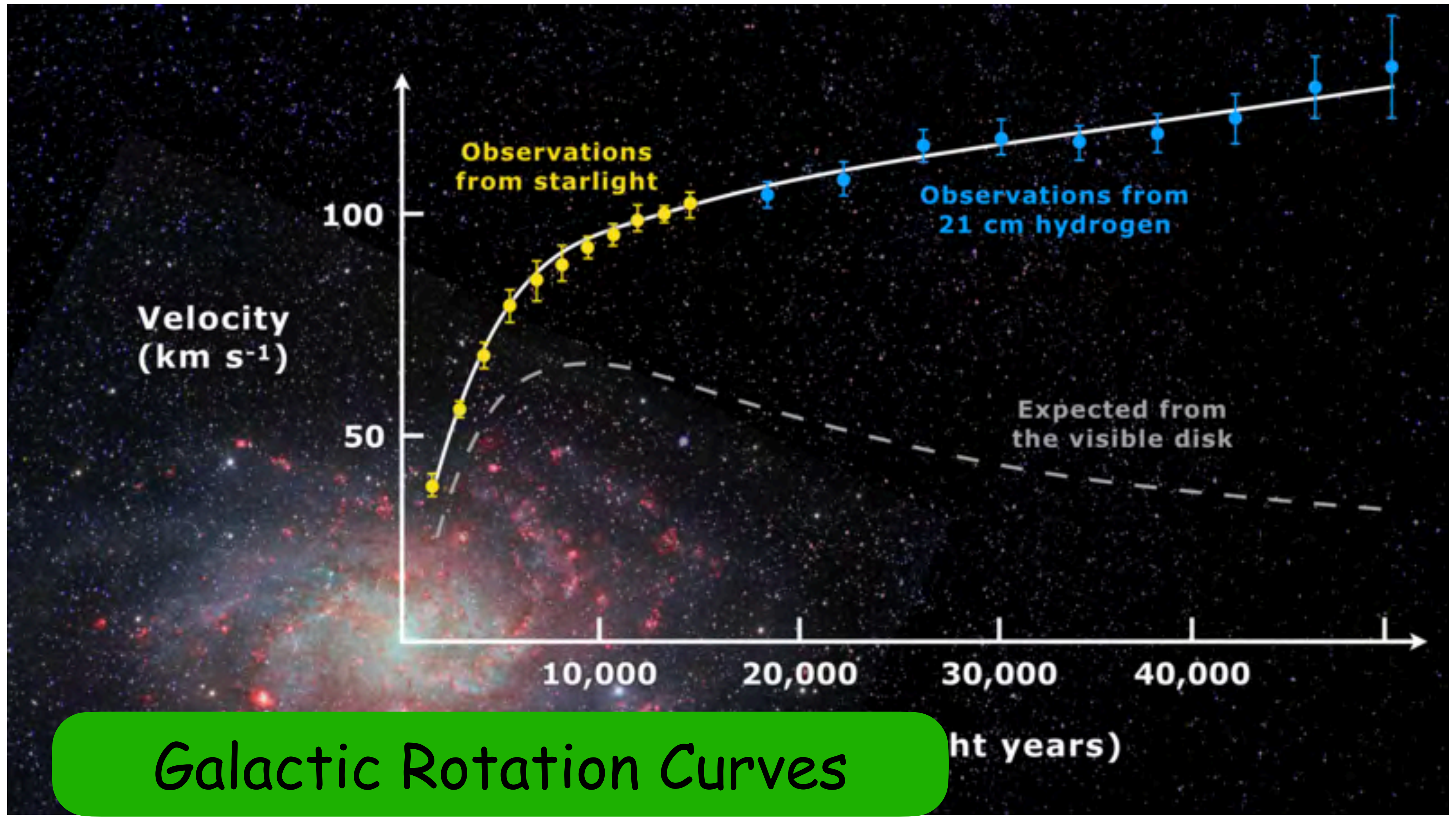


UC San Diego

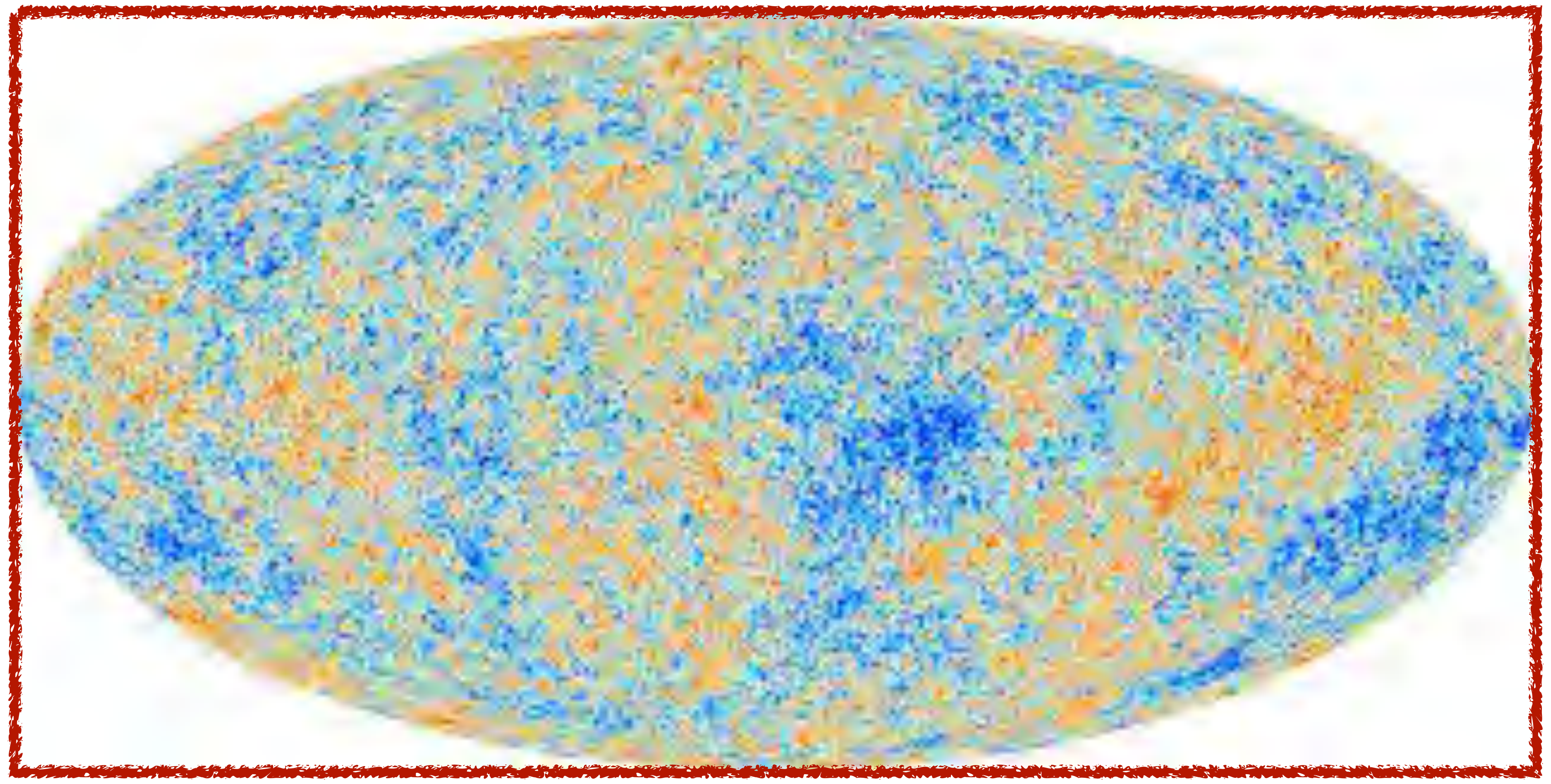
RSC, Dennis Foren, Joshua Andrew Gill, Kirtimaan Mohan, George Sanamyan, Dipan Sengupta, Xing Wang, EHS

Part 1: Dark Matter

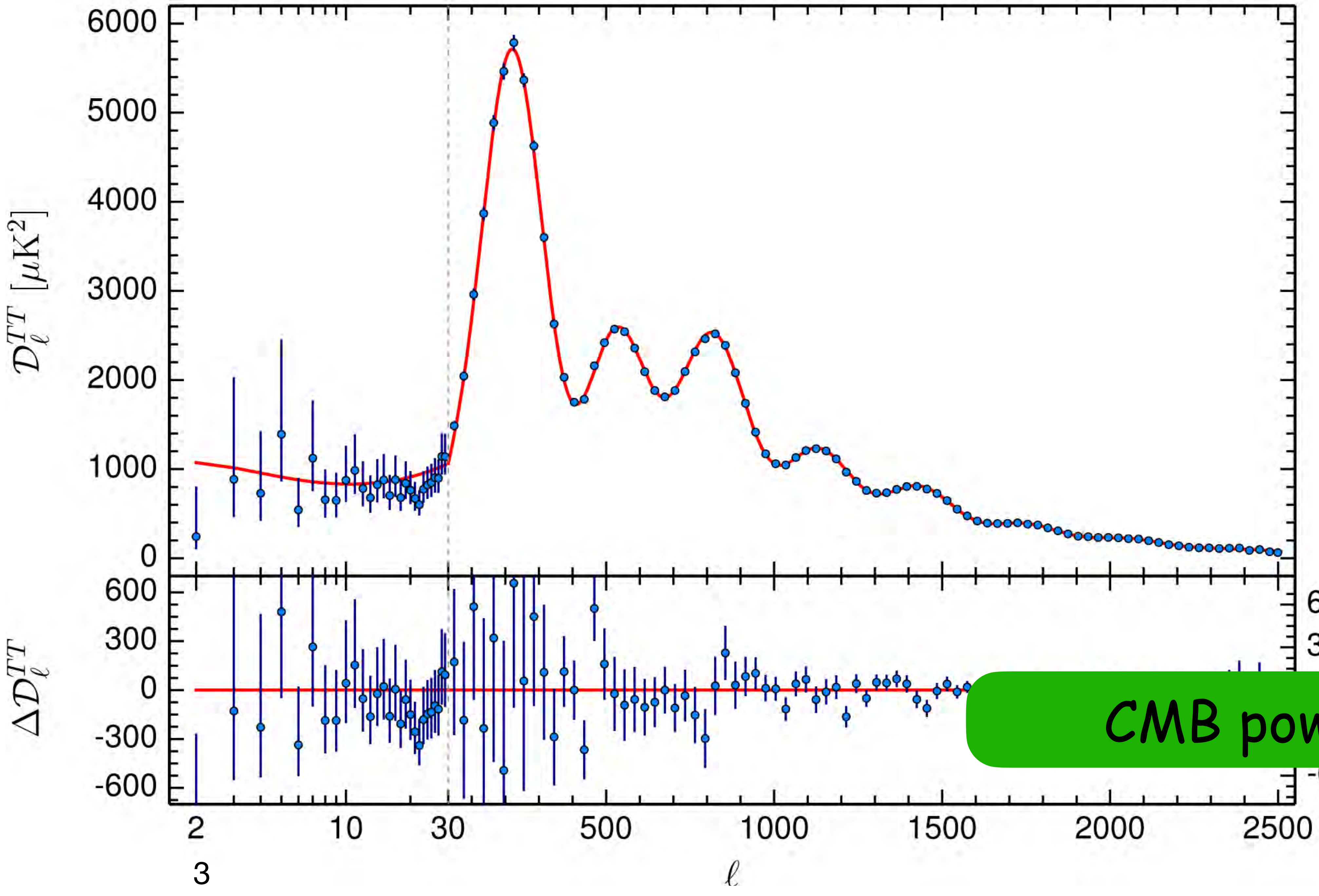
Dark Matter Evidence



Galactic Rotation Curves

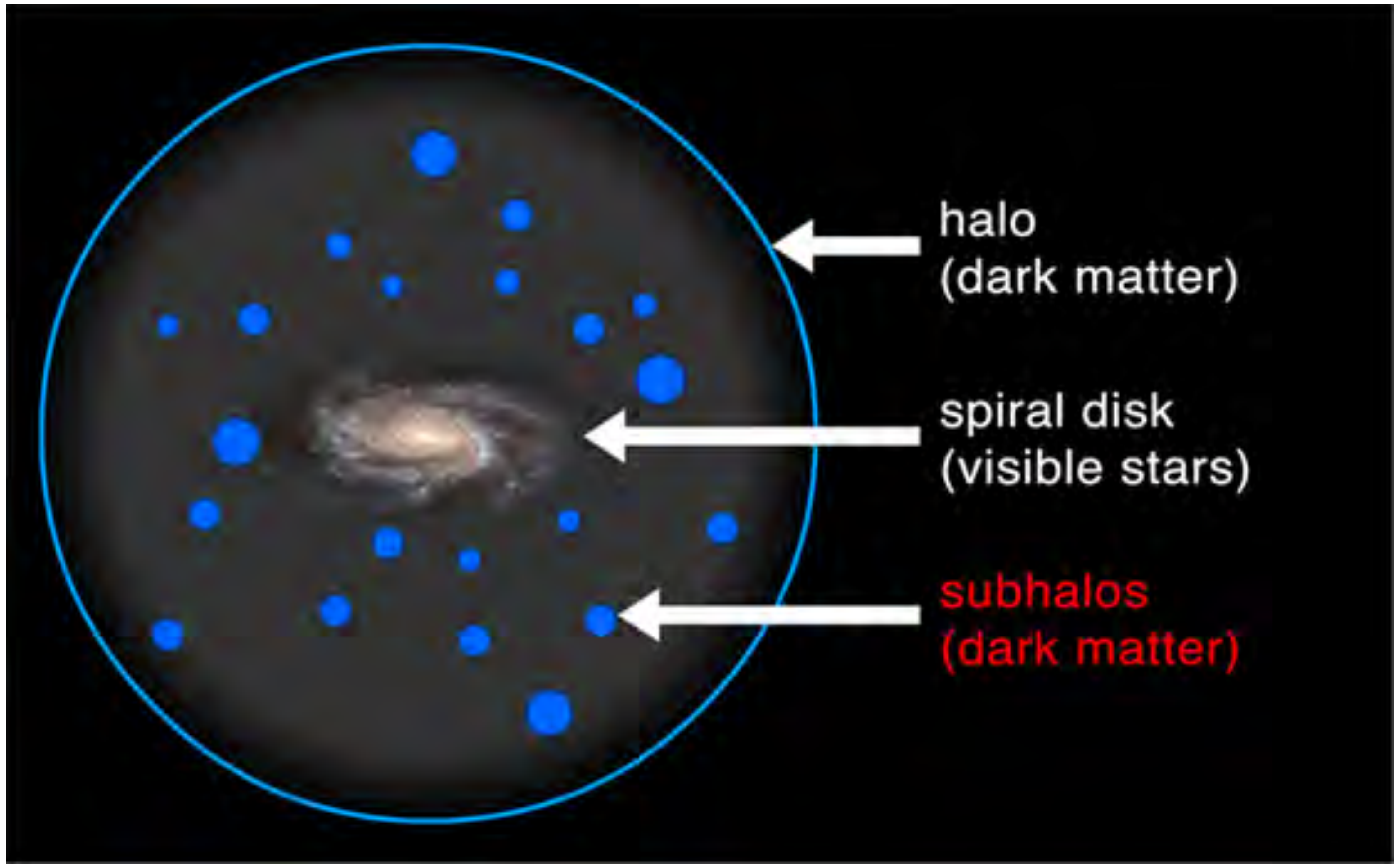


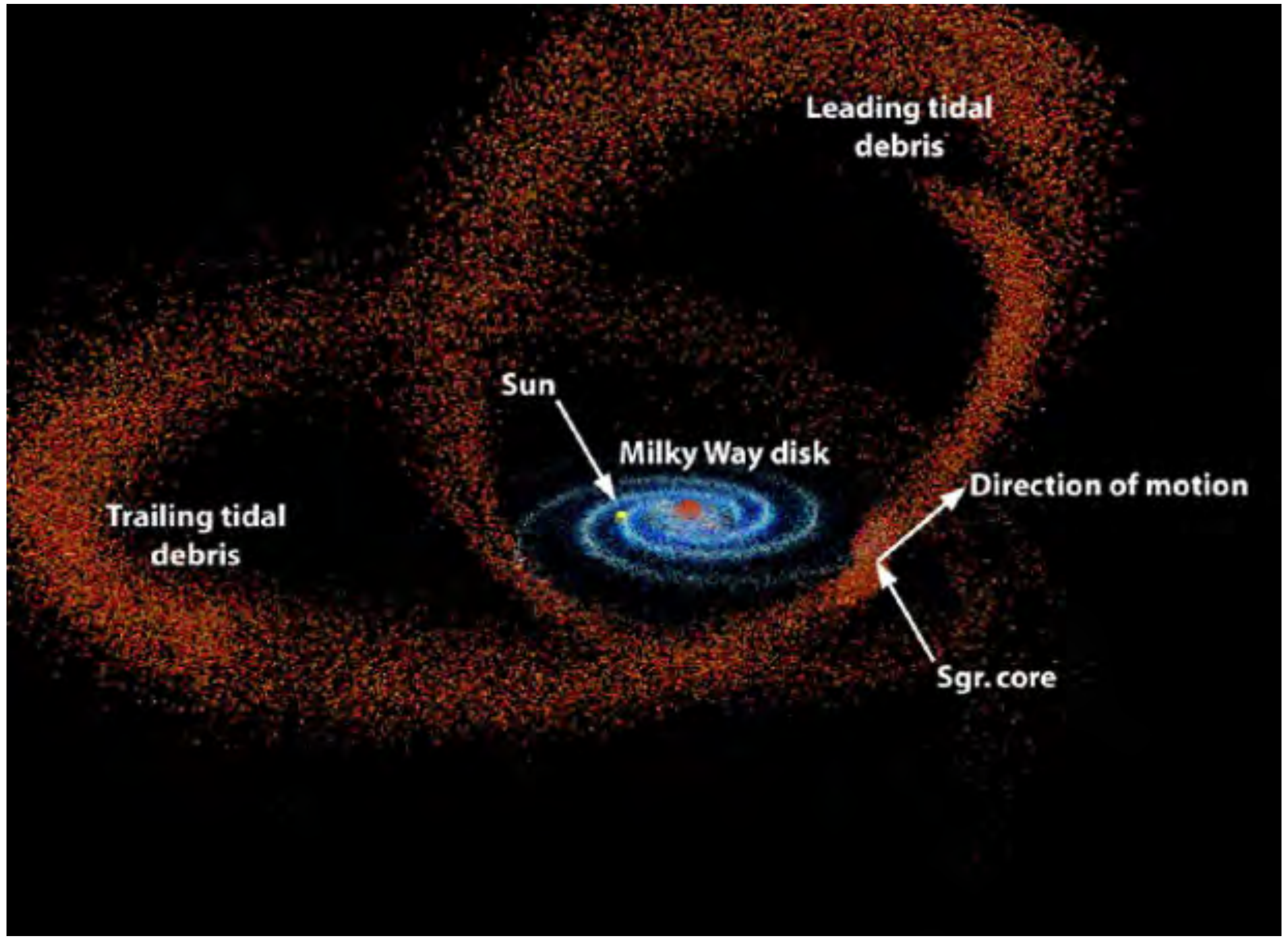
Gravitational Lensing in Bullet Cluster .
Pink- X-Ray image
Blue : Gravitational lensing image



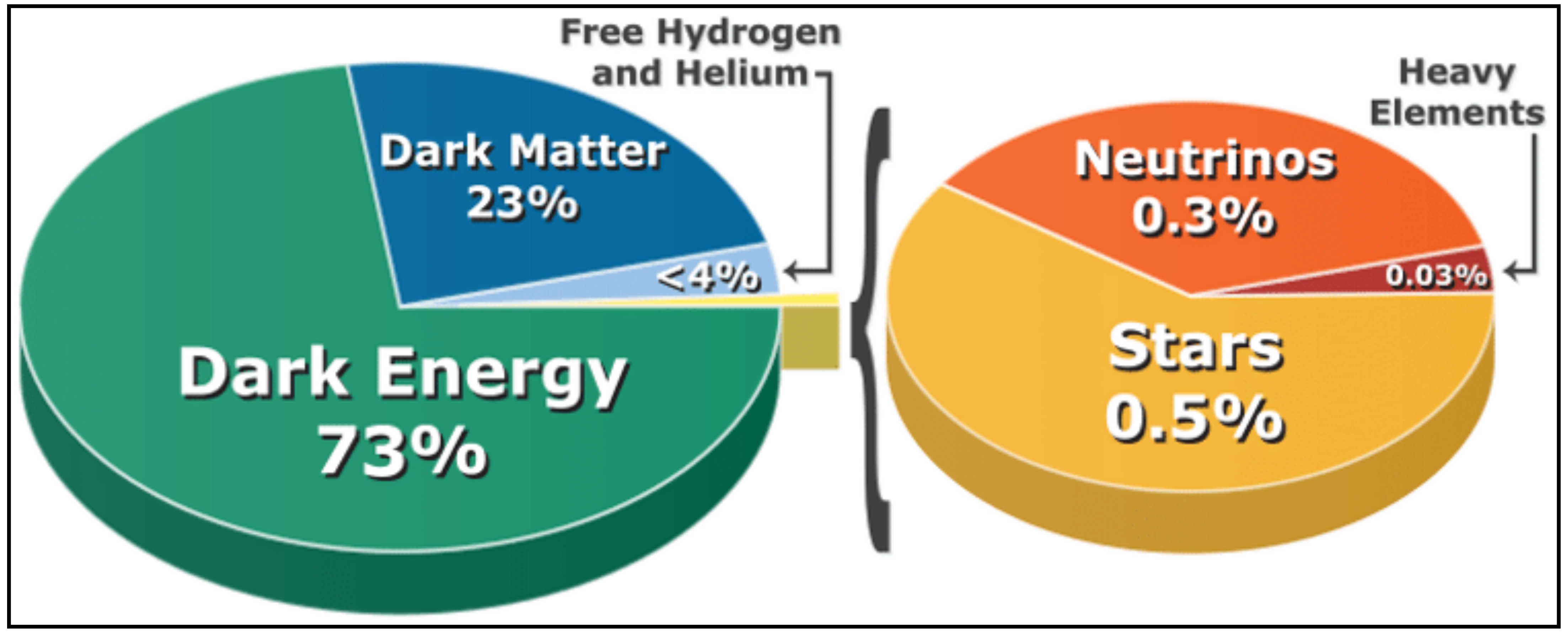
CMB power spectrum

Properties and the Particle Physics of Dark Matter





- Cold and Neutral: Non relativistic today.
- Preserves the success of Big Bang Nucleosynthesis (Formation of Atoms and Nuclei in the early Universe)
- "Almost" **Dark** with respect to other forces of nature.
- Collisionless within the DM sector at large scales.
- Stable, on Cosmological time scales.
- Forms halos in the galaxy

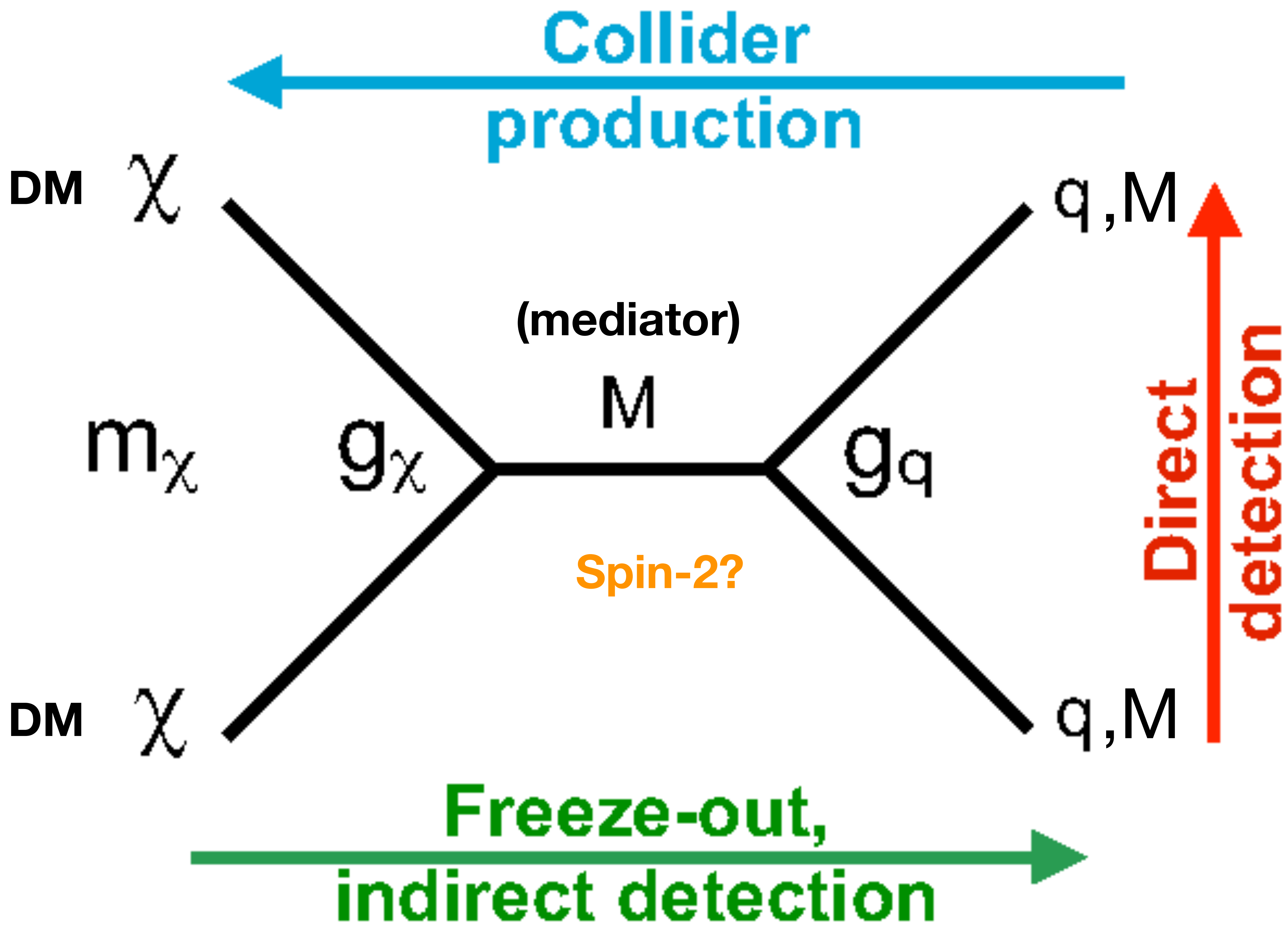


What particle possible particle explanations are there for dark matter?

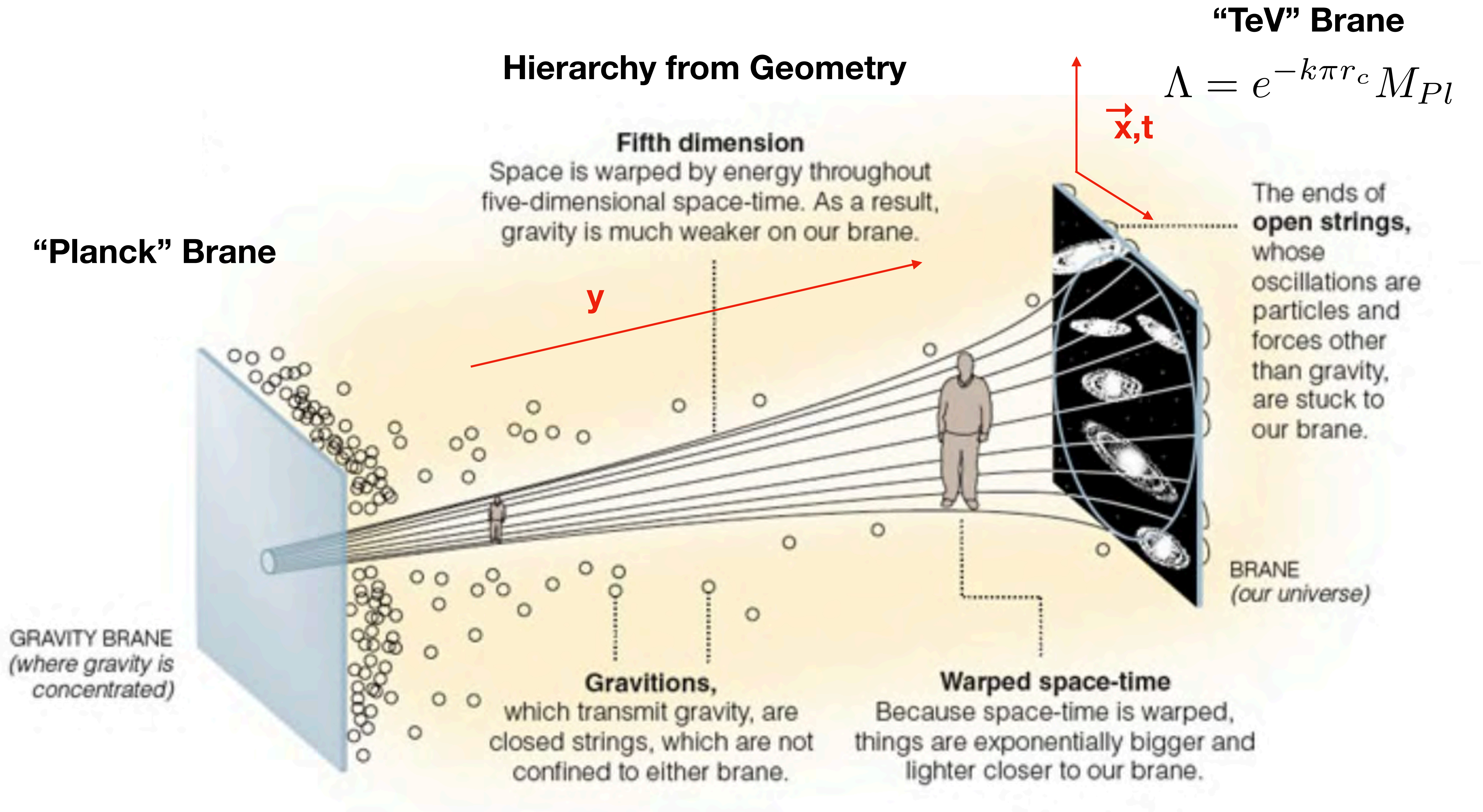
Dark Matter Models

Spin-2 KK Portal Dark Matter

Spin-2 Mediator



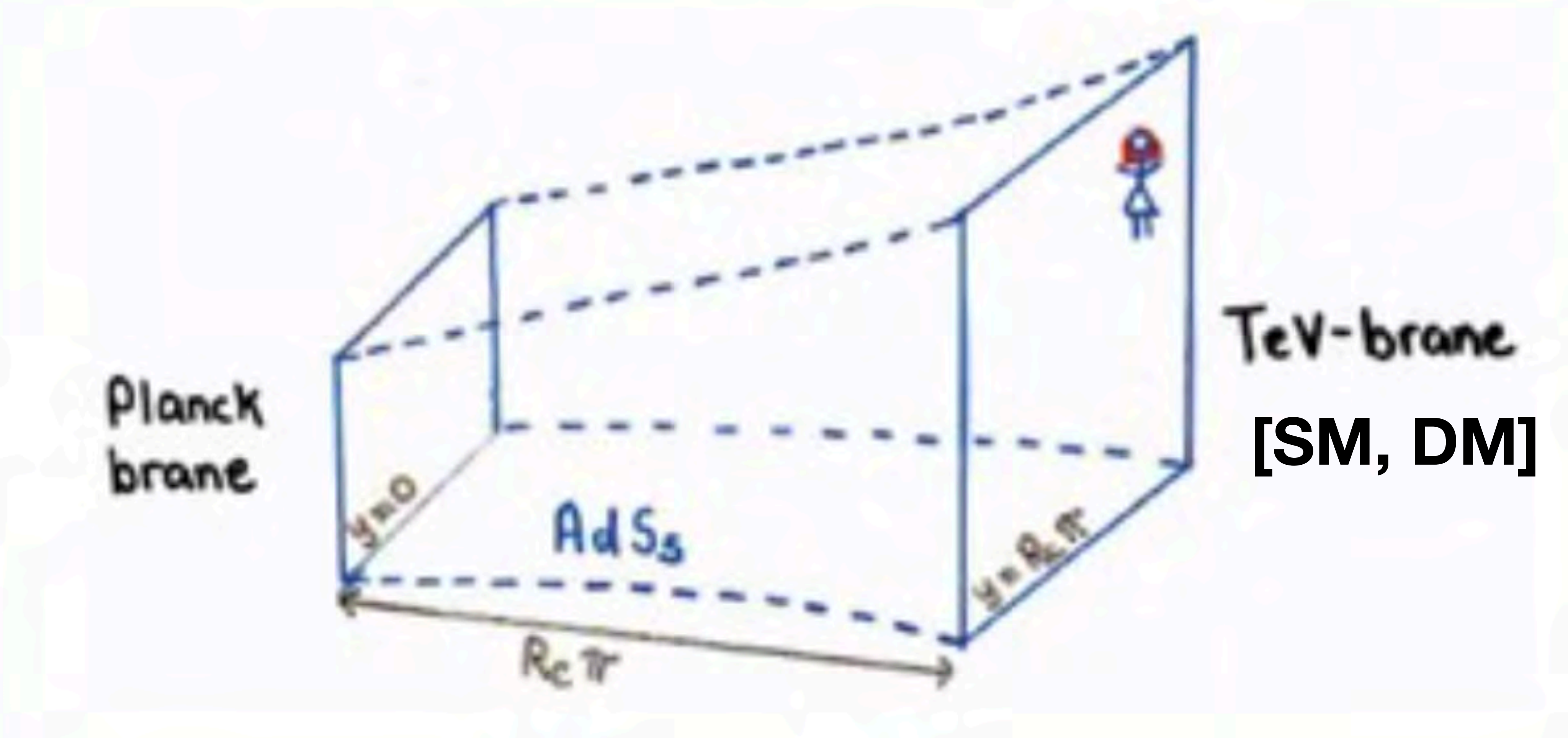
Randall Sundrum (RS-I) Model



$$ds^2 = e^{-2k|y|}(dt^2 - d\vec{x}^2) - dy^2$$

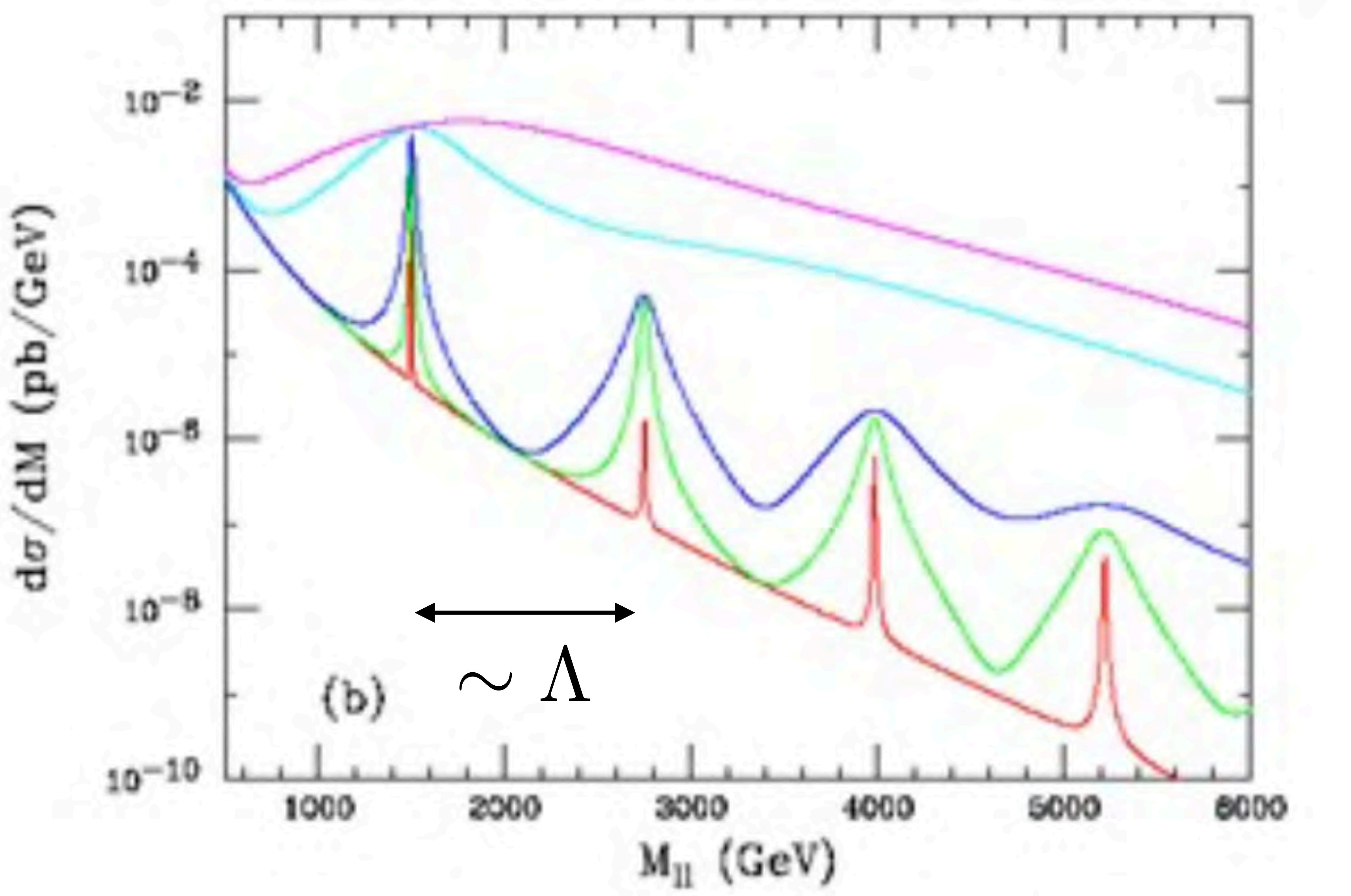
Kaluza-Klein Gravitons

Gravity propagates in the finite bulk



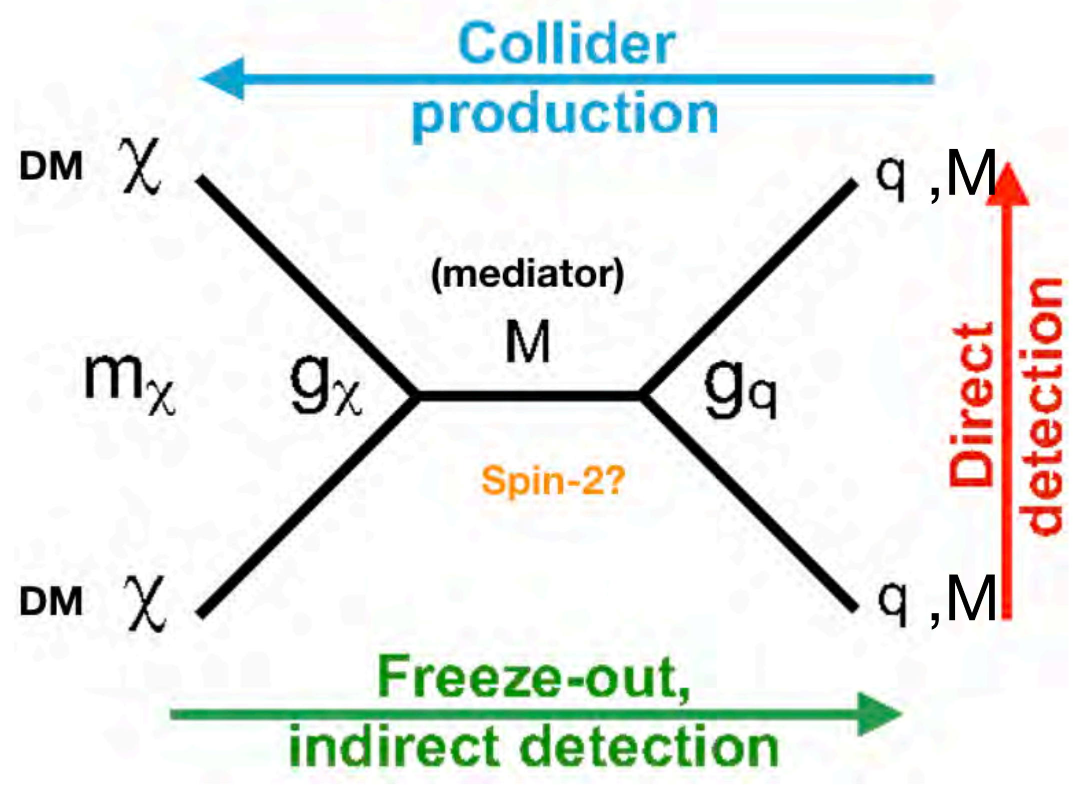
$$\Lambda = e^{-k\pi r_c} M_{Pl}$$

Potential LHC Resonances…

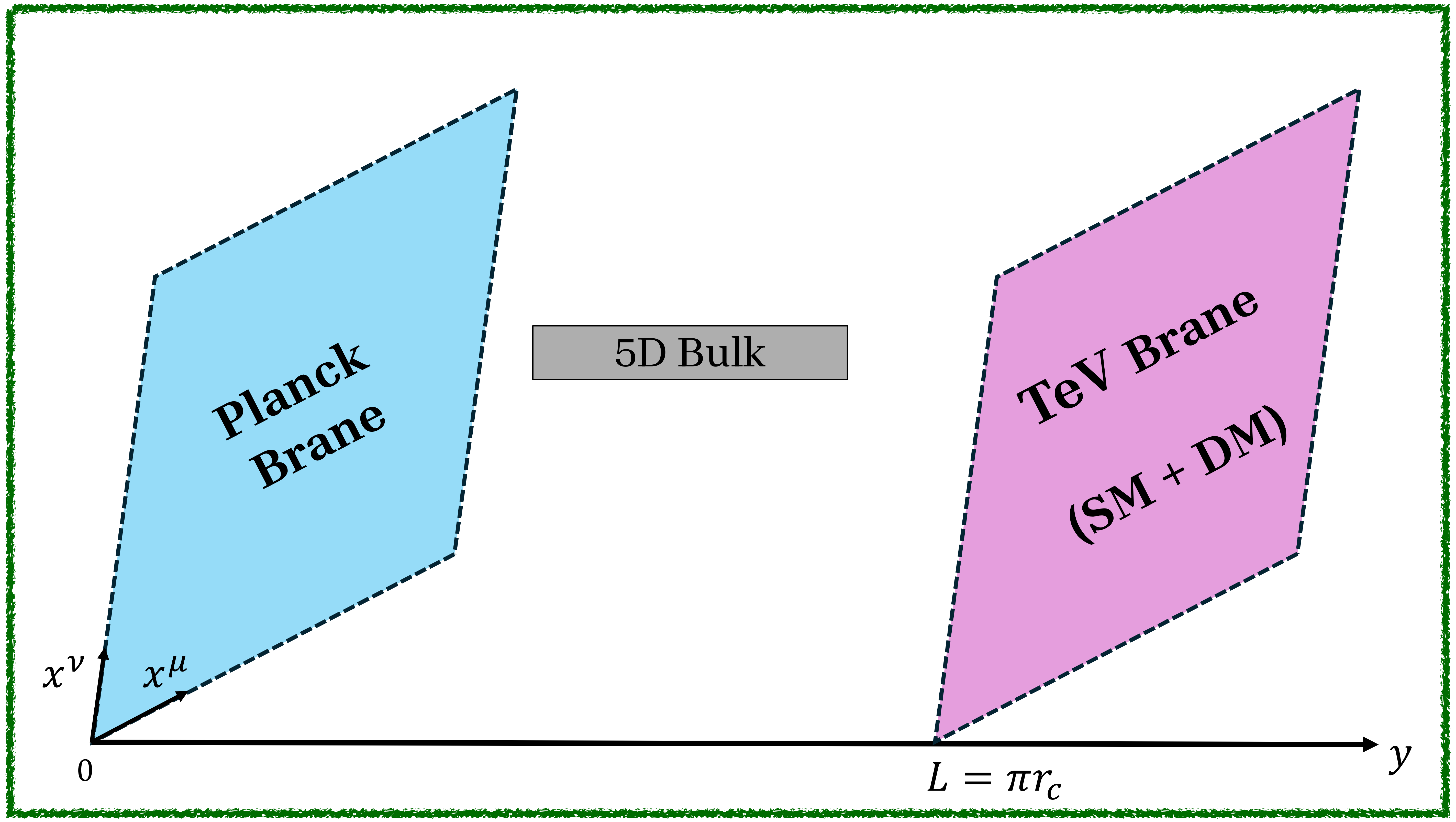


Discrete KK modes: RS Gravitons

$$\mathcal{L}_{int} = \frac{1}{\Lambda}\sum_i c_i X_i^{\mu\nu}(T_{\mu\nu}^{SM} + T_{\mu\nu}^{DM})$$



Extra-Dimensional Models with KK portals

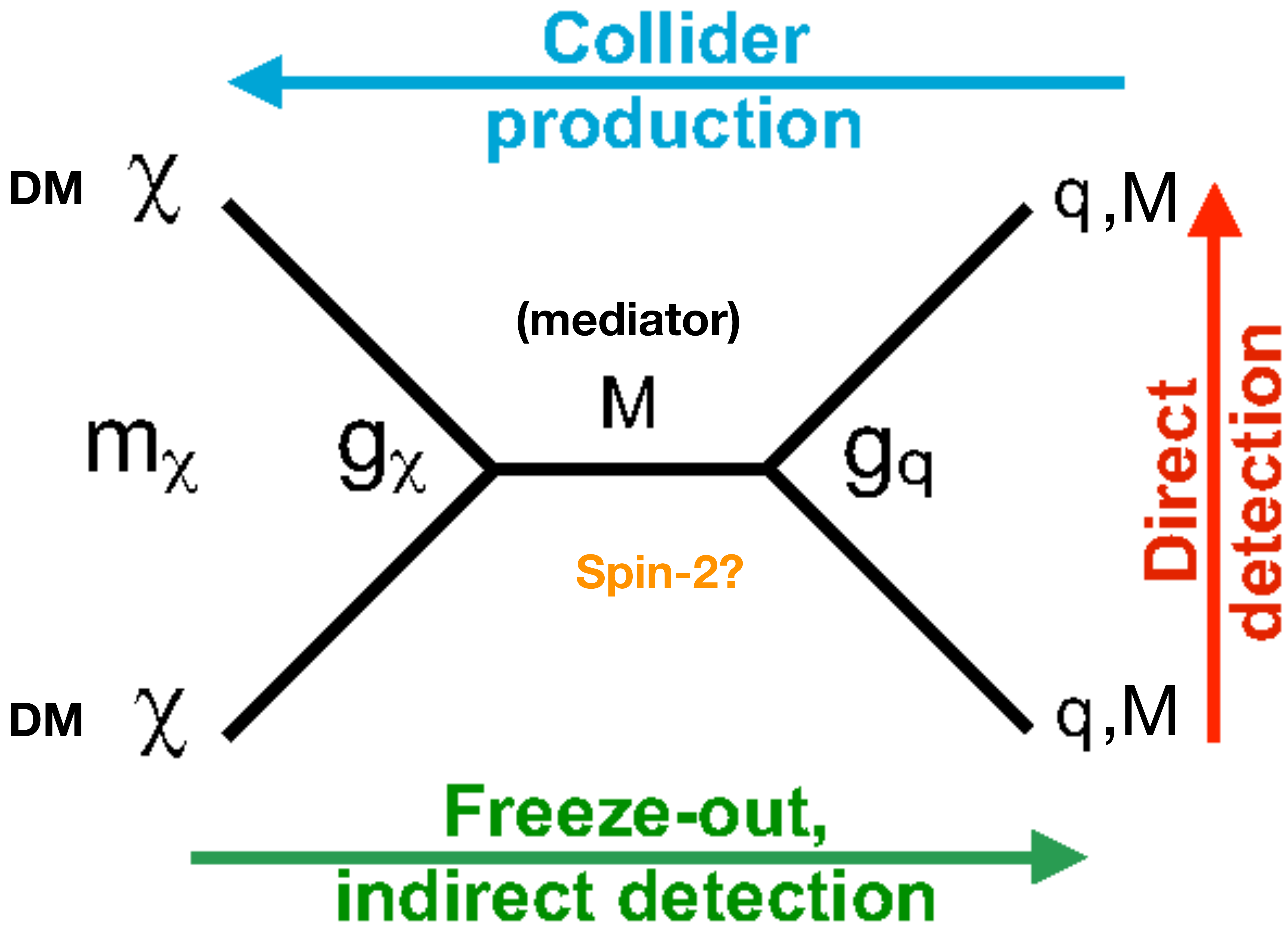


Dark Sector interacts with visible sector exclusively via gravitational interactions from the KK sector

Reuter, Rizzo, Hewett 2017, Sanz et al, 2017, Garny et al 2017, Olive, Mambrini et al 2019 .....

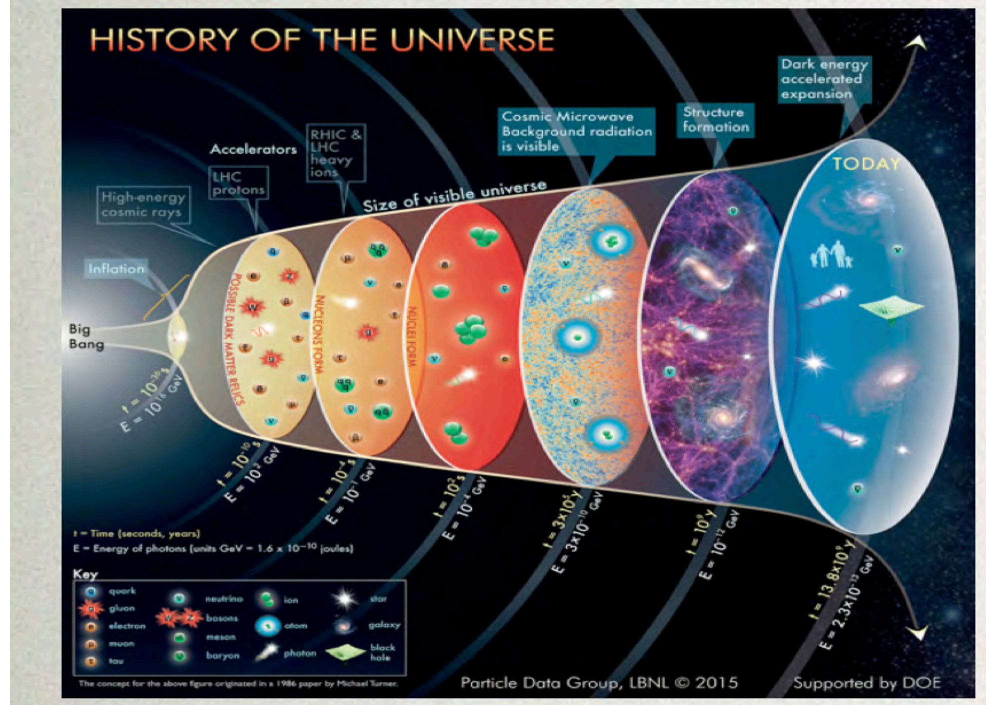
KK Portal Dark Matter Production: Thermal Freeze-Out

Spin-2 Mediator



Production of Dark Matter in the Early Universe

A THERMAL ORIGIN



Simple and predictive Boltzmann equation governs evolution of number density "n"

$$\frac{dn}{dt} = -3Hn - \langle \sigma_A v \rangle \left(n^2 - n_{\mathrm{eq}}^2 \right)$$

As Universe cools below DM mass, density decreases as $e^{-m/T}$

Eventually dark matter particles can't find each other to annihilate

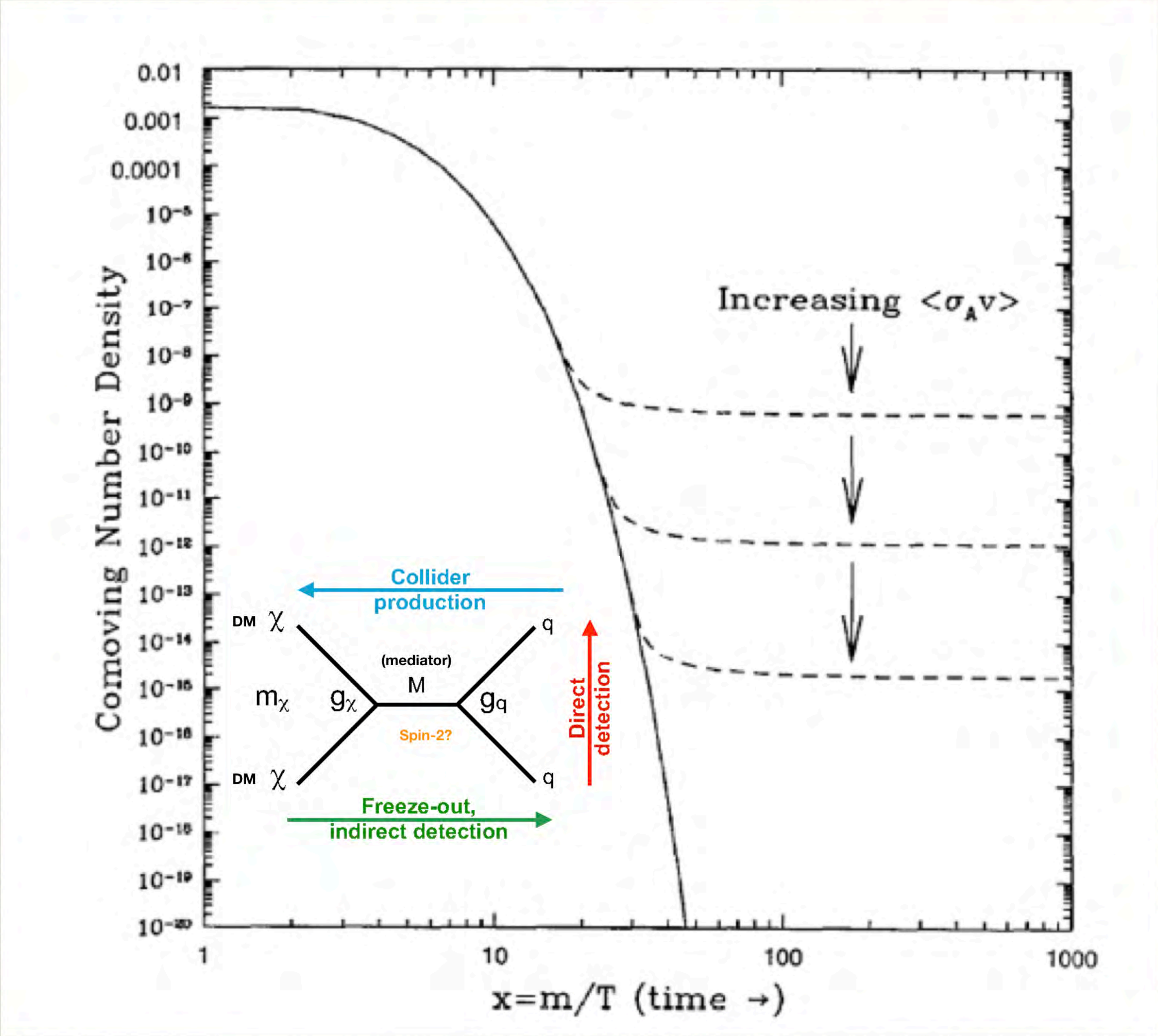
$$\longrightarrow \quad n \langle \sigma_A v \rangle = H$$

freeze-out occurs

Schuster, 2019 FNAL DM Workshop

Thermal Freeze-Out Cross Section

WIMPs and a Thermal Origin



Larger cross-section
⇒ later freeze-out
⇒ lower density

Correct DM density for:

$$\langle \sigma v \rangle \simeq 3 \times 10^{-26}\ \mathrm{cm}^3/\mathrm{s}$$

$$\simeq \frac{1}{(20\ \mathrm{TeV})^2}$$

Schuster, 2019 FNAL DM Workshop

However: correctly computing the massive spin-2 scattering amplitudes is hard!

Brane Localized Scalar Dark Matter annihilating via a KK/massive spin-2 portal

Gravity-mediated Scalar Dark Matter in Warped Extra-Dimensions

Miguel G. Folgado,[a] Andrea Donini,[a] Nuria Rius[a]

PRL **116,** 101302 (2016) PHYSICAL REVIEW LETTERS week ending 11 MARCH 2016

Planckian Interacting Massive Particles as Dark Matter

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(Received 20 November 2015; published 10 March 2016)

PHYSICAL REVIEW LETTERS **128,** 081806 (2022)

Massive Gravitons as Feebly Interacting Dark Matter Candidates

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Spin-2 portal dark matter

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Dark Matter Direct Detection from new interactions in models with spin-two mediators

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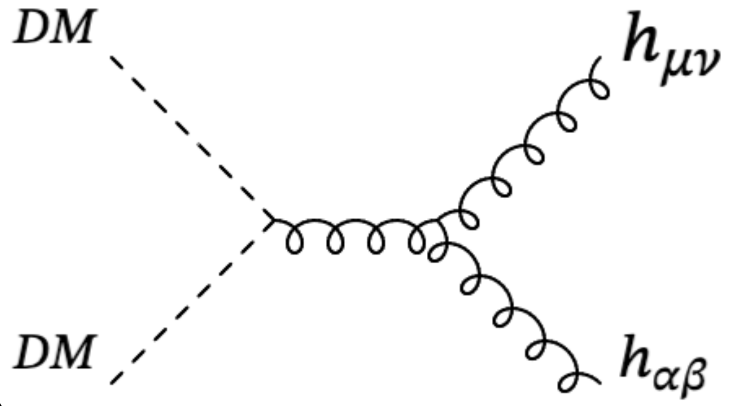
Naive (wrong) scaling of KK-annhilation:

$$\boxed{|\mathcal{M}| = \mathcal{O}\left(\frac{E^6}{m^4 M_{Pl}^2}\right)}$$



In fact, diffeomorphism invariance is restored in a full Kaluza-Klein theory and $|\mathcal{M}| = \mathcal{O}\left(\frac{E^2}{M_{Pl}^2}\right)$

Phys.Rev.D 100 (2019) 11, 115033 *Phys.Rev.D* 101 (2020) 7, 075013 *Phys.Rev.D* 101 (2020) 5, 055013 **Phys. Rev. D 103 (2022), 095024**

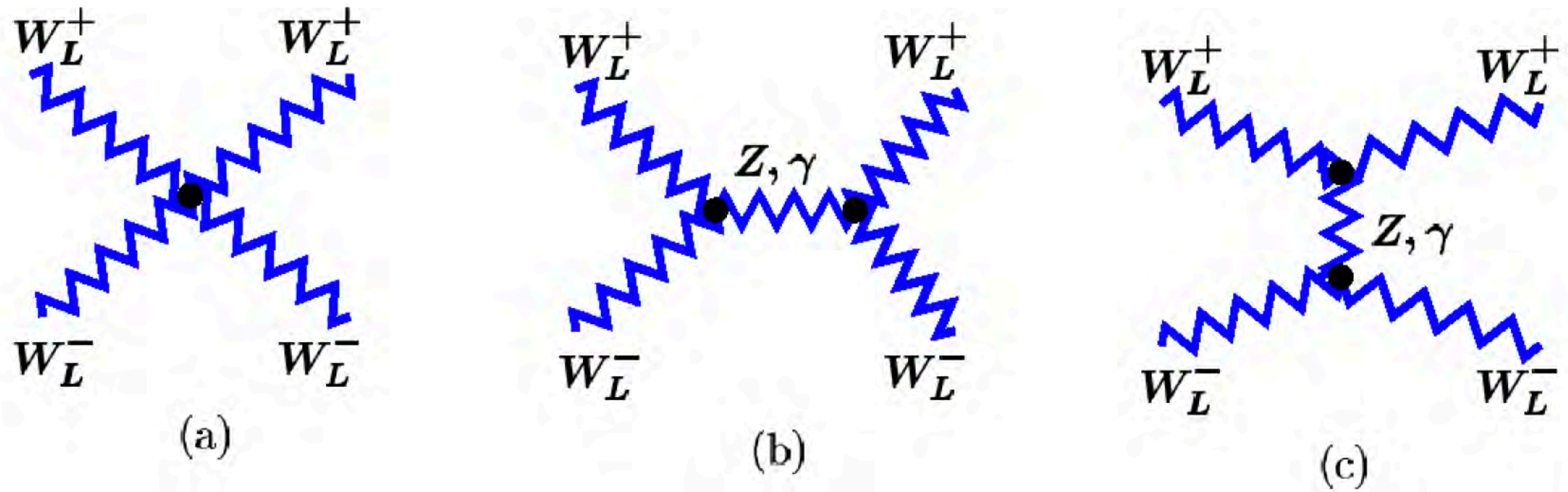
Phys. Rev. D 107,(2023) 03505, PRD (submitted)

R. S. Chivukula, J.A.Gill, K. Mohan, G. Sanmyan, D. Sengupta, E.H. Simmons, X. Wang

Part 2: Scattering Amplitudes of Massive Spin-2 Particles

Reminder: Massive Spin-1 Scattering Amplitudes in the SM and KK Yang-Mills Theories

Weinberg-Glashow-Salam: SU(2) x U(1) @ E^4

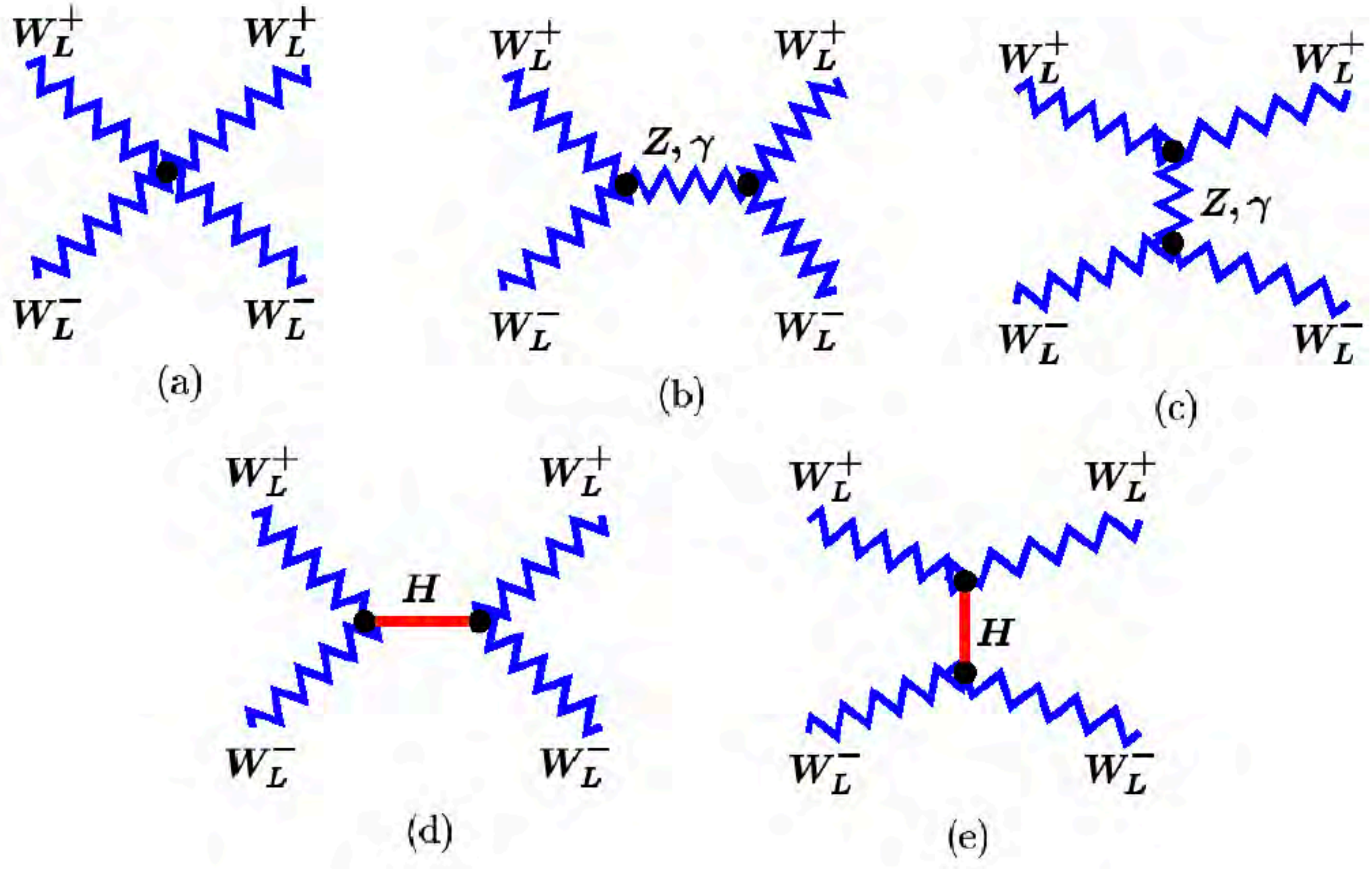


Graphs	$g^2 \frac{E^4}{m_w^4}$
(a)	$-3 + 6\cos\theta + \cos^2\theta$
(b)	$-4\cos\theta$
(c)	$+3 - 2\cos\theta - \cos^2\theta$
Sum	0

E^4 divergences cancel due to YM Jacobi Identity (symmetry structure)

$$\epsilon_L^\mu(k) = \frac{k^\mu}{m_w} + \mathcal{O}\left(\frac{m_w}{E}\right)$$

Weinberg-Glashow-Salam: SU(2) x U(1) @ E²



Graphs	$g^2 \frac{E^2}{m_w^2}$
(a)	$+2 - 6\cos\theta$
(b)	$-\cos\theta$
(c)	$-\frac{3}{2} + \frac{15}{2}\cos\theta$
(d + e)	$-\frac{1}{2} - \frac{1}{2}\cos\theta$
Sum including (d+e)	**0**

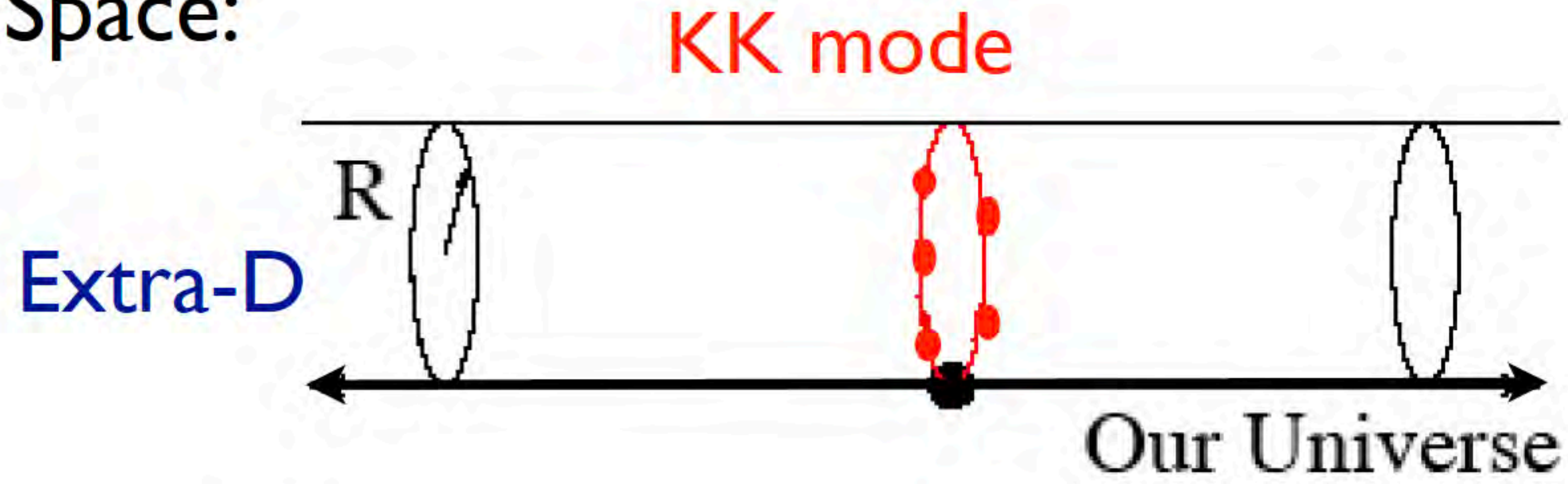
- $\mathcal{O}(E^0) \Rightarrow$ 4d m_H bound: $m_H < \sqrt{16\pi/3}\, v \simeq 1.0$ TeV
- If no Higgs $\Rightarrow \mathcal{O}(E^2) \Rightarrow$ $E < \sqrt{4\pi}\, v \simeq 0.9$ TeV

Higgs boson couplings to gauge-bosons such that
E² growth cancels between all contributions!

Lee, Quigg, Thacker

Five-Dimensional Yang-Mills: Spin-1 KK Modes

Flat Space:



Expand 5-D gauge bosons in eigenmodes: e.g. for S^1/Z_2:

$$\hat{A}^a_\mu = \frac{1}{\sqrt{\pi R}}\left[A^{a0}_\mu(x_\nu) + \sqrt{2}\sum_{n=1}^{\infty} A^{an}_\mu(x_\nu)\cos\left(\frac{n x_5}{R}\right)\right]$$

$$\hat{A}^a_5 = \sqrt{\frac{2}{\pi R}}\sum_{n=1}^{\infty} A^{an}_5(x_\nu)\sin\left(\frac{n x_5}{R}\right)$$

"eaten" Goldstone Bosons

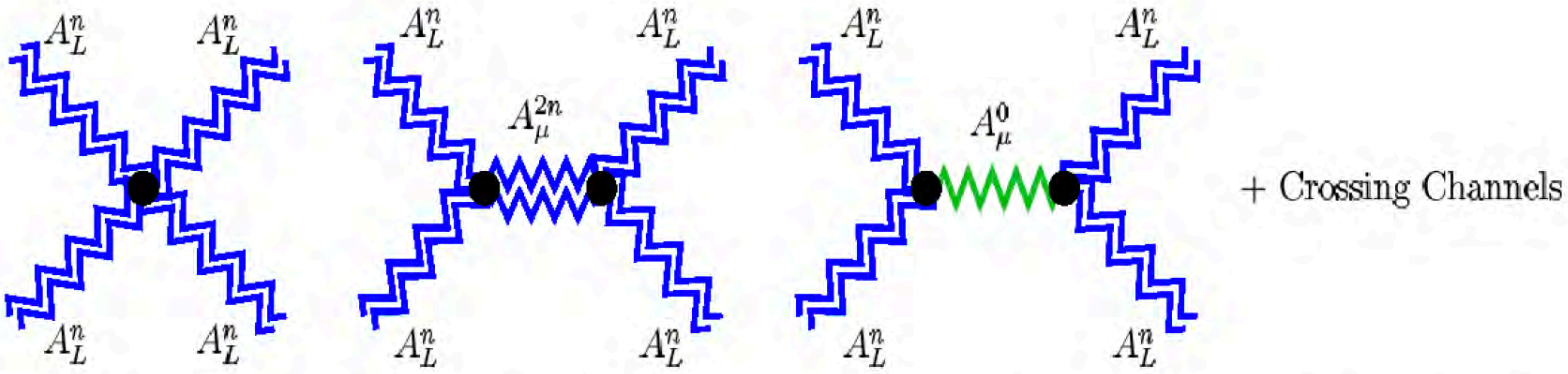
4-D gauge kinetic term contains

$$M_n = \frac{n}{R}$$

$$\frac{1}{2}\sum_{n=1}^{\infty}\left[M_n^2 (A^{an}_\mu)^2 - 2 M_n A^{an}_\mu \partial^\mu A^{an}_5 + (\partial_\mu A^{an}_5)^2\right]$$

i.e., $A^{an}_L \leftrightarrow A^{an}_5$

4-D KK Mode Scattering



(a) $(b1)$ $(c1)$ $(b2, b3)+(c2, c3)$

Cancellation of bad high-energy behavior through exchange of massive vector particles

graph	$g^2 C^{eab} C^{ecd}$	$g^2 C^{eac} C^{edb}$	$g^2 C^{ead} C^{ebc}$
(a)	$6c(x^4 - x^2)$	$\frac{3}{2}(3-2c-c^2)x^4$ $-3(1-c)x^2$	$\frac{-3}{2}(3+2c-c^2)x^4$ $+3(1+c)x^2$
$(b1)$	$-2c(x^4 \perp x^2)$		
$(c1)$	$-4cx^4$		
$(b2,3)$		$\frac{-1}{2}(3-2c+c^2)x^4$ $+3(1-c)x^2$	$\frac{1}{2}(3+2c-c^2)x^4$ $-3(1+c)x^2$
$(c2,3)$		$(-3+2c+c^2)x^4$ $-8cx^2$	$(3+2c-c^2)x^4$ $-8cx^2$
Sum	$-8cx^2$	$-8cx^2$	$-8cx^2 \Rightarrow \mathbf{0}$

$$\mathcal{O}\left(\frac{g^2 E^2}{M^2}\right)$$

RSC, H.J. He, D. Dicus (2002)
Csaki, Grojean, Murayama, Pilo, Terning (2004)

Generalizations …

- Applies to any 5-D manifold with Lorentz Invariance for fixed x^5: $$A^\mu = \sum_n A_n^\mu(x^\nu) f_n(x^5)$$
- KK eigenvalues and wavefunctions depend on metric and boundary conditions
- KK couplings related to overlap of KK wavefunctions: $$g_{nmp} \propto \int dx^5\, f_n(x^5)\, f_m(x^5)\, f_p(x^5)$$
- Yields 4-D Lorentz-invariant effective field theory for infinite tower of KK modes.

Important related paper: "Hidden" SUSY guarantees that "eaten" A_5 modes have masses correctly related to KK A_μ masses!

Supersymmetry in gauge theories with extra dimensions

C.S. Lim (Kobe U.), Tomoaki Nagasawa (Kobe U.), Makoto Sakamoto (Kobe U.), Hidenori Sonoda (Kobe U.)

(Feb, 2005)

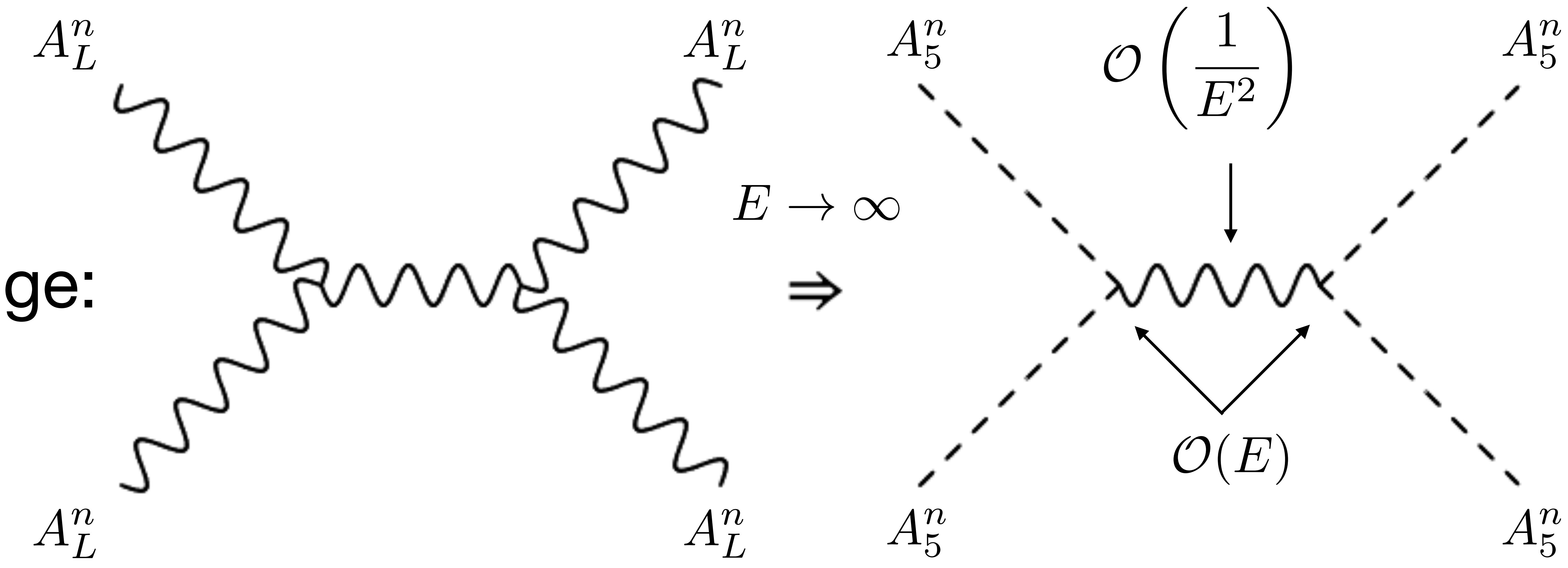
Published in: *Phys.Rev.D* 72 (2005) 064006 • e-Print: hep-th/0502022 [hep-th]

Symmetry Constrains Scattering Amplitudes: Equivalence Theorem

Gauge-Invariance of 5D YM theory implies we can choose `t-Hooft-Feynman Gauge

- Rewrite theory including “unphysical” Goldstone (scalar) bosons, in “A_5” field
- Problematic longitudinal helicity amplitudes are, to leading order in energy, the same as those of the Goldstone bosons, by gauge-invariance
- Propagator and interactions such every diagram scales like $O(E^0)$
- Hence, physical (unitary gauge) scattering amplitudes must also scale like $O(E^0)$!

’t-Hooft-Feynman gauge:



Interacting Massive Spin-2 Particles

Massive Spin-2 Particles and Degrees of Freedom

- Massless graviton represented by a symmetric traceless tensor with redundant degrees of freedom (local diffeomorphism invariance)

- In general a massless graviton in d dimensions has d(d-3)/2 physical degrees of freedom

$$h_{\mu\nu} \to h_{\mu\nu} + \partial_{(\mu}\xi_{\nu)}$$

  - Massless 4D graviton has 2 degrees of freedom, while a massless 5D graviton has 5 degrees of freedom.

- Mass term breaks diffeomorphism invariance - A 4D massive spin-2 state has 5 degrees of freedom (2 spin-2 helicity states, 2 spin-1 helicity states and 1 spin-0 helicity state)

$$m^2((h_{\mu\nu})^2 - h^2)$$

Fierz-Pauli Mass term (ghost-free in flat space)

- **A compactified 5D theory of gravity, therefore, can provide the states needed for the 4D massive KK spin-2 modes.**

Massive Spin-2 Helicity States and (Unitary Gauge) Propagator

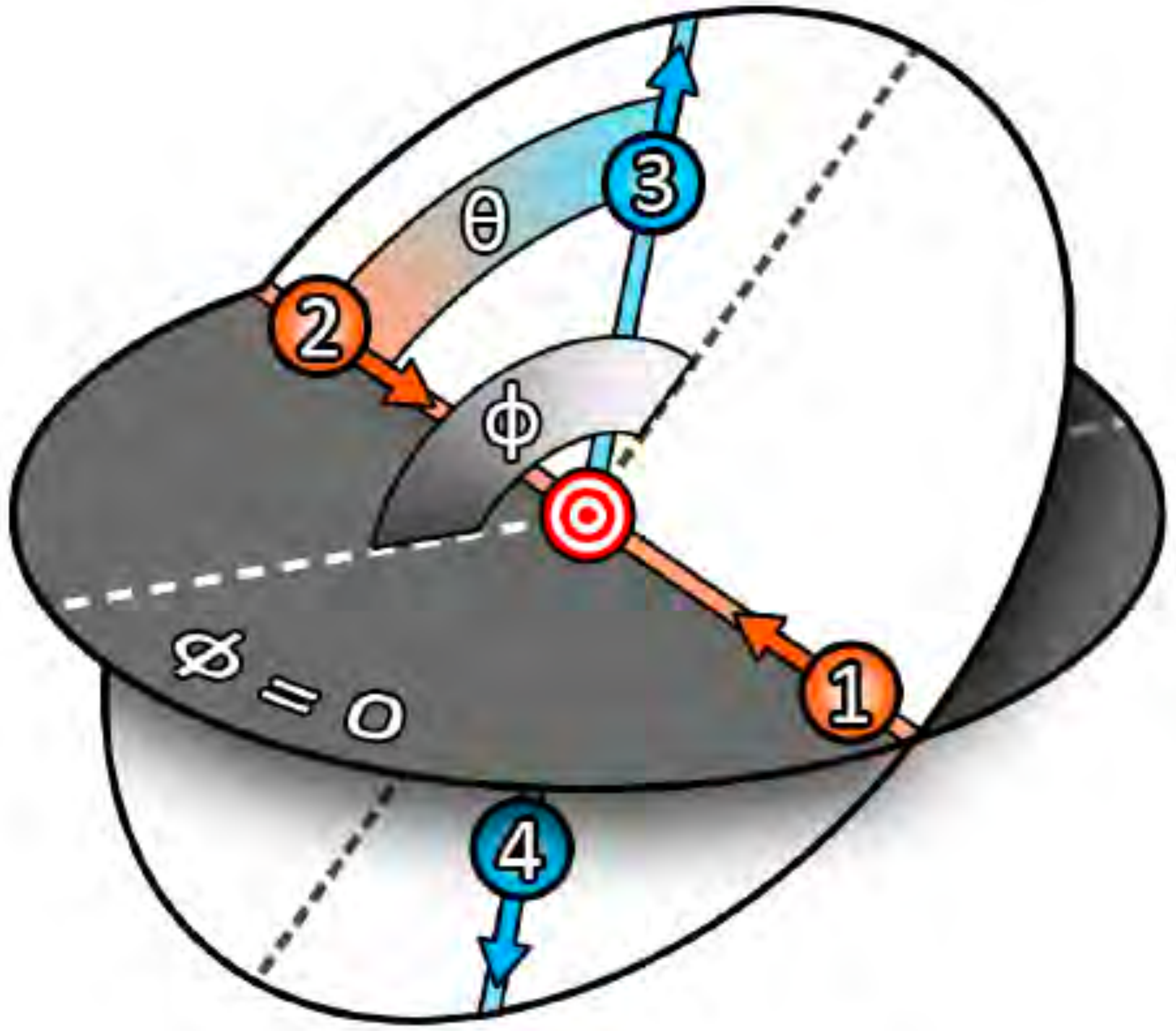
Initial state

$$p^\mu_{i_1} = (E_{i_1}, +|\vec{p}_i|\hat{z}) \qquad p^2_{i_1} = m^2_k$$
$$p^\mu_{i_2} = (E_{i_2}, -|\vec{p}_i|\hat{z}) \qquad p^2_{i_2} = m^2_l$$

Final state

$$p^\mu_{f_1} = (E_{f_1}, +\vec{p}_f) \qquad p^2_{f_1} = m^2_m$$
$$p^\mu_{f_2} = (E_{f_2}, -\vec{p}_f) \qquad p^2_{f_2} = m^2_n$$

$$\vec{p}_f \equiv |\vec{p}_f|(\sin\theta\cos\phi, \sin\theta\sin\phi, \cos\theta).$$



Polarization tensors

Spin-2 longitudinal

$$\epsilon^{\mu\nu}_0 = \frac{1}{\sqrt{6}}\left[\epsilon^\mu_{+1}\epsilon^\nu_{-1} + \epsilon^\mu_{-1}\epsilon^\nu_{+1} + 2\epsilon^\mu_0\epsilon^\nu_0\right]$$

Spin-1

$$\epsilon^\mu_{\pm 1} = \frac{e^{\pm i\phi}}{\sqrt{2}}\left[\mp\partial_\theta - \frac{i}{\sin\theta}\partial_\phi\right]\epsilon^\mu_0$$
$$\epsilon^\mu_0 = \frac{1}{m}\left(\sqrt{E^2 - m^2}, E\,\hat{p}\right)$$

Propagator

$$\frac{iB^{\mu\nu,\rho\sigma}}{P^2 - M^2} \qquad B^{\mu\nu,\rho\sigma} \equiv \frac{1}{2}\left[\overline{B}^{\mu\rho}\overline{B}^{\nu\sigma} + \overline{B}^{\nu\rho}\overline{B}^{\mu\sigma} - \frac{1}{3}(2+\delta_{0,M})\overline{B}^{\mu\nu}\overline{B}^{\rho\sigma}\right] \qquad \overline{B}^{\alpha\beta} \equiv \eta^{\alpha\beta} - \frac{P^\alpha P^\beta}{M^2}\delta_{0,M}$$

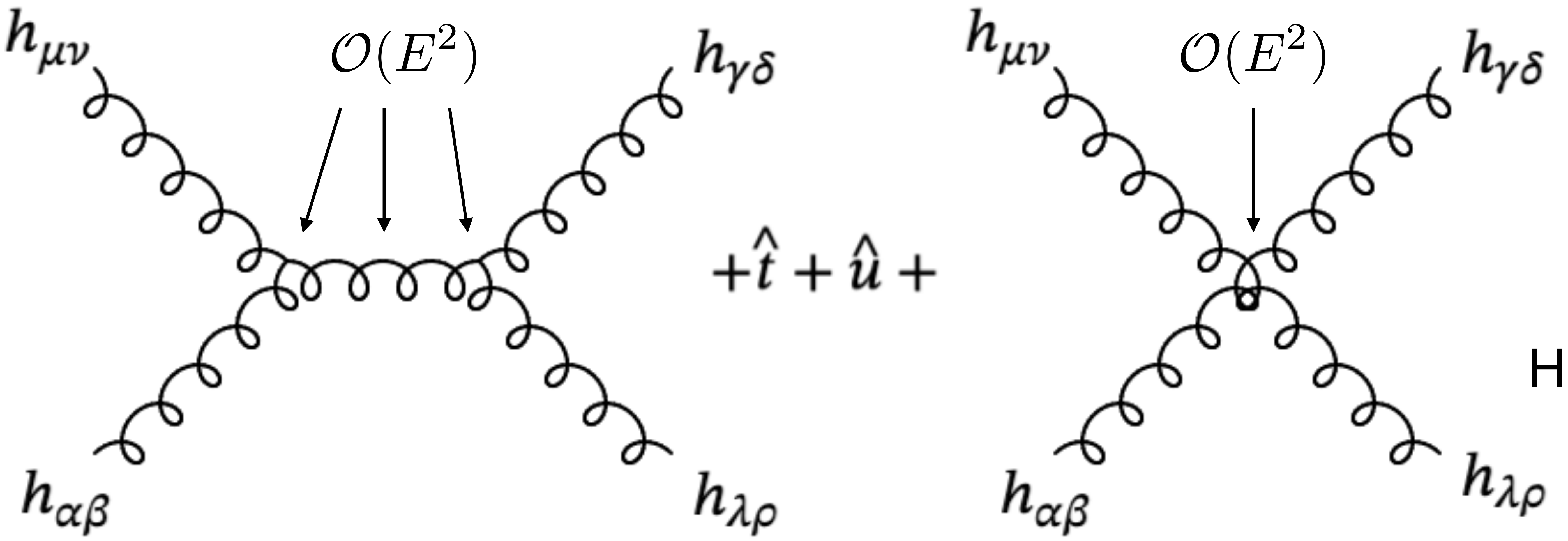
Unitary Gauge …

(absent when M=0)

Mandelstam s

$$s \equiv (p_{i_1} + p_{i_2})^2 = (E_{i_1} + E_{i_2})^2$$

Elastic Scattering of Helicity-0 Fierz-Pauli Massive Spin-2 Particles



$$= \mathcal{O}\left(\frac{E^{10}}{m^8 M_{Pl}^2}\right)$$

Helicity-0

$$g_{\mu\nu} \to \eta_{\mu\nu} + \kappa h_{\mu\nu}$$

$$S_G = \frac{2}{\kappa^2}\int d^4x\sqrt{-g}R$$

$$+ \; m^2((h_{\mu\nu})^2 - h^2)$$

Helicity-0 polarization at high-energy: $\varepsilon_0^{\mu\nu} \Rightarrow \dfrac{k^\mu k^\nu}{m^2}$

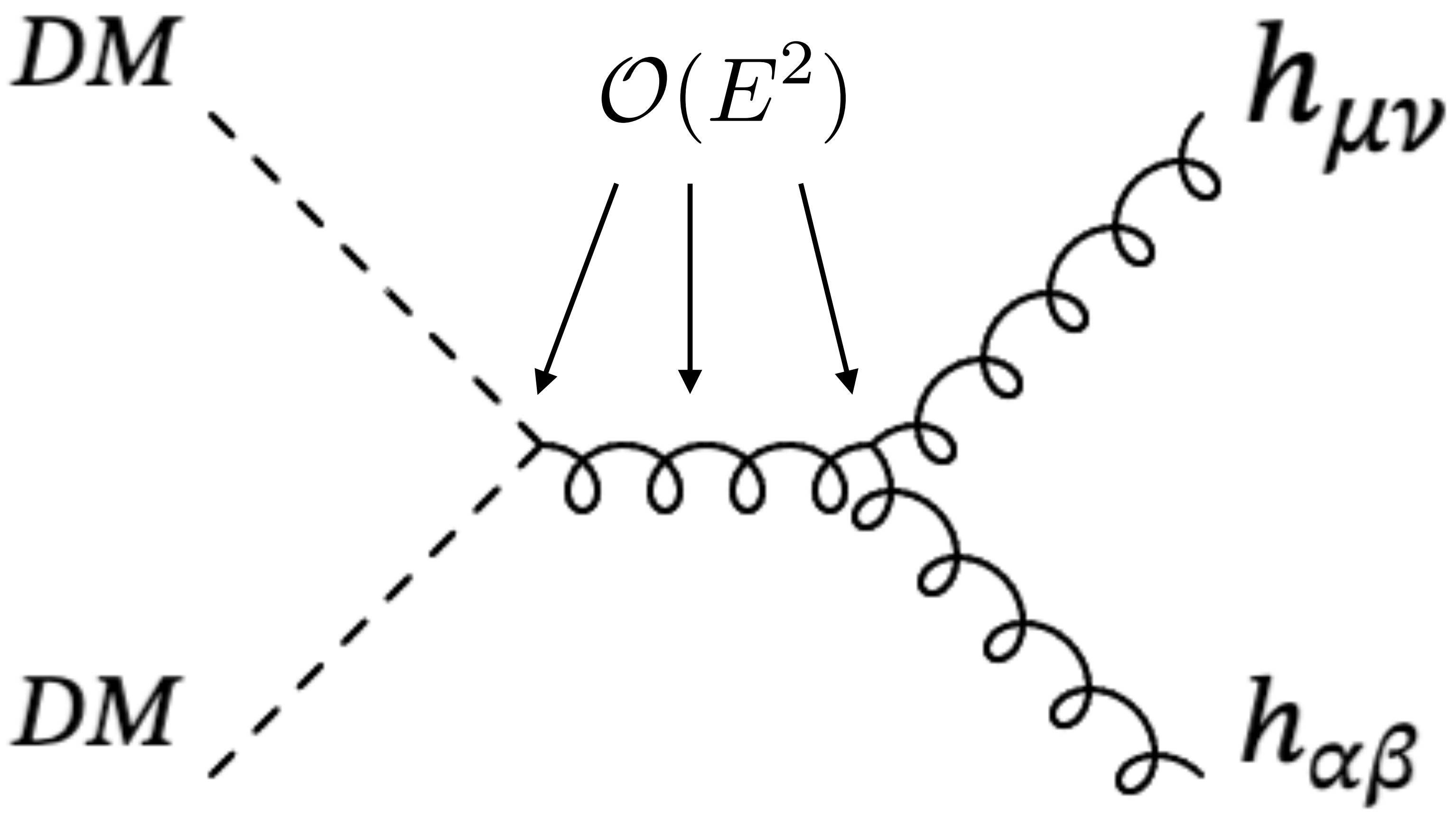
Propagator

$$\frac{iB^{\mu\nu,\rho\sigma}}{P^2 - M^2} \qquad B^{\mu\nu,\rho\sigma} \equiv \frac{1}{2}\left[\overline{B}^{\mu\rho}\overline{B}^{\nu\sigma} + \overline{B}^{\nu\rho}\overline{B}^{\mu\sigma} - \frac{1}{3}(2+\delta_{0,M})\overline{B}^{\mu\nu}\overline{B}^{\rho\sigma}\right] \qquad \overline{B}^{\alpha\beta} \equiv \eta^{\alpha\beta} - \frac{P^\alpha P^\beta}{M^2}$$

Unitary Gauge …

Unitarity is violated at a scale $\Lambda_5 = (M_{pl}m^4)^{1/5} \ll M_{pl}$

Massive Spin-2 Fierz-Pauli Scattering to DM: Naive Expectation



$$= \mathcal{O}\left(\frac{E^6}{m^4 M_{Pl}^2}\right)$$

Propagator

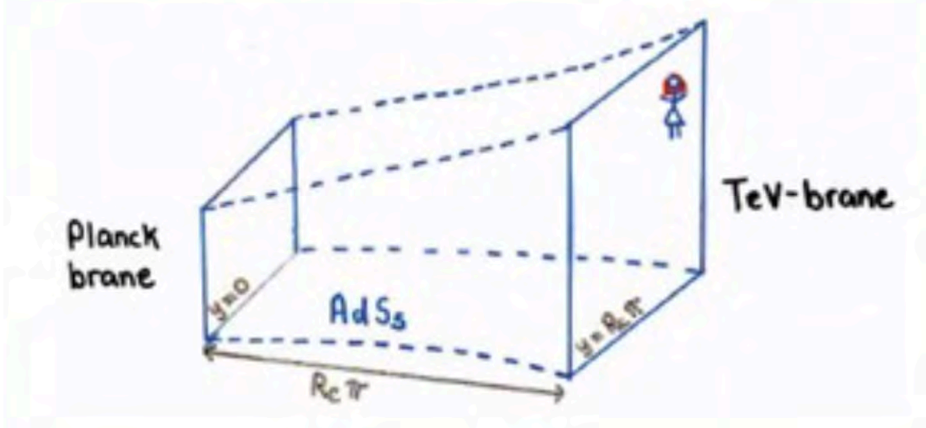
$$\frac{iB^{\mu\nu,\rho\sigma}}{P^2 - M^2} \qquad B^{\mu\nu,\rho\sigma} \equiv \frac{1}{2}\left[\overline{B}^{\mu\rho}\overline{B}^{\nu\sigma} + \overline{B}^{\nu\rho}\overline{B}^{\mu\sigma} - \frac{1}{3}(2+\delta_{0,M})\overline{B}^{\mu\nu}\overline{B}^{\rho\sigma}\right] \qquad \overline{B}^{\alpha\beta} \equiv \eta^{\alpha\beta} - \frac{P^\alpha P^\beta}{M^2}$$

Unitary Gauge …

Spin-2 RS1 KK Scattering Amplitudes

Randall Sundrum Model

Many options for perturbing vacuum. The **Einstein frame parameterization** is automatically canonical in 4D:



$$G_{MN} = \begin{pmatrix} e^{-2(k|y|+\hat{u})}(\eta_{\mu\nu}+\kappa\hat{h}_{\mu\nu}) & 0 \\ 0 & -(1+2\hat{u})^2 \end{pmatrix} \qquad \hat{u} \equiv \frac{\kappa\,\hat{r}}{2\sqrt{6}} e^{+k(2|y|-\pi r_c)}$$

where κ characterizes the perturbation and $[\kappa] = [\text{Energy}]^{-3/2}$

Particles in 5D Matter-Free Randall-Sundrum Model:

- **5D Graviton** $= \hat{h}_{\mu\nu}$, a massless spin-2 5D particle
  ▷ **Origin:** local coordinate invariance of constant y sheets

- **5D Radion** $= \hat{r}$, a massless spin-0 5D particle
  ▷ **Origin:** locally perturbing distance between branes

The situation in a KK theory is different due to the underlying 5D diffeomorphism invariance

RS Fields and Interactions

Once we have a 5D theory, we convert it into an effective 4D theory via y-integration and **Kaluza-Klein decomposition**:

$$\boxed{\mathcal{L}_{4\text{D}}^{(\text{RS,eff})} \equiv \int_{-\pi r_c}^{+\pi r_c} dy\ \mathcal{L}_{5\text{D}}^{(\text{RS})}}$$

$$\boxed{\underbrace{\hat{f}_{\vec{\mu}}(x,y)}_{\text{5D Field}} = \frac{1}{\sqrt{\pi r_c}} \sum_{n=0}^{+\infty} \underbrace{\hat{f}_{\vec{\mu}}^{(n)}(x)}_{\text{4D Fields}}\ \underbrace{\psi^{(n)}(y)}_{\text{Wfxn}}}$$

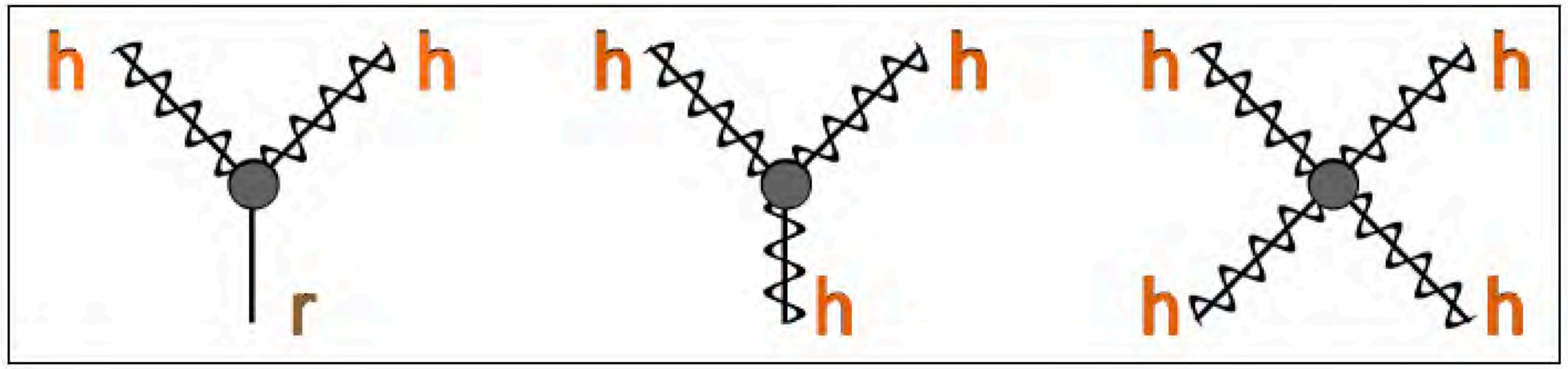
Define $\mathcal{L}_{h^H r^R}^{(\text{RS})} \equiv$ all terms in $\mathcal{L}_{5\text{D}}^{(\text{RS})}$ with H graviton fields and R radion fields. By construction, each term in this set is either

- **A-Type:** has two spatial derivatives $\partial_\mu \partial_\nu$, or
- **B-Type:** has two extra-dimensional derivatives ∂_y^2

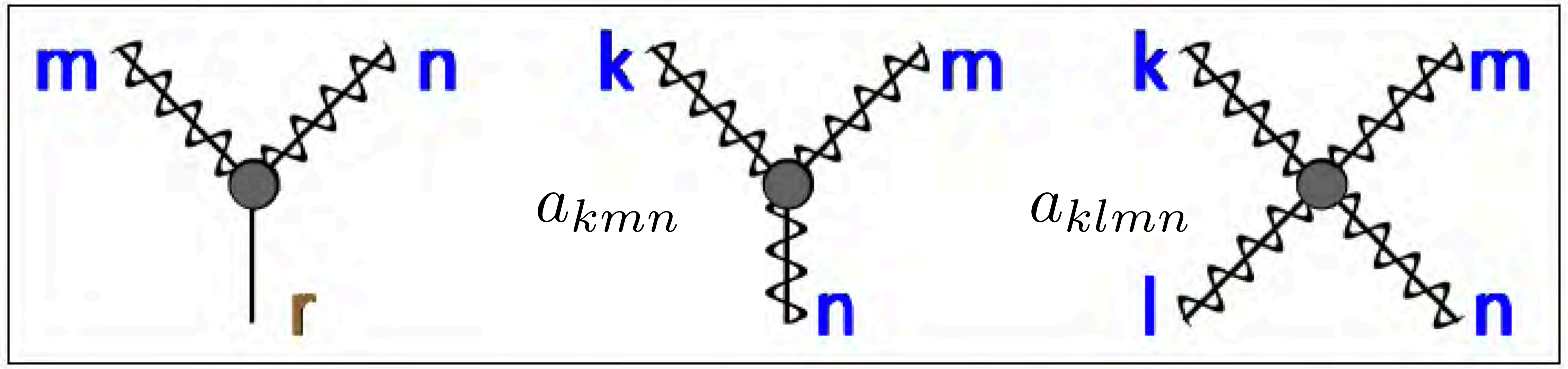
$$\begin{aligned}\mathcal{L}_{h^H r^R}^{(\text{RS})} &= \mathcal{L}_{A:h^H r^R}^{(\text{RS})} + \mathcal{L}_{B:h^H r^R}^{(\text{RS})} \\ &= \kappa^{(H+R-2)} \left[e^{k[2(R-1)|y| - R\pi r_c]}\, \overline{\mathcal{L}}_{A:h^H r^R}^{(\text{RS})} + e^{k[2(R-2)|y| - R\pi r_c]}\, \overline{\mathcal{L}}_{B:h^H r^R}^{(\text{RS})} \right]\end{aligned}$$

RS KK Interactions

$\mathcal{L}_{5D}^{(RS)}$ contains the following important vertices:



which then imply 4D effective vertices via KK decomposition:

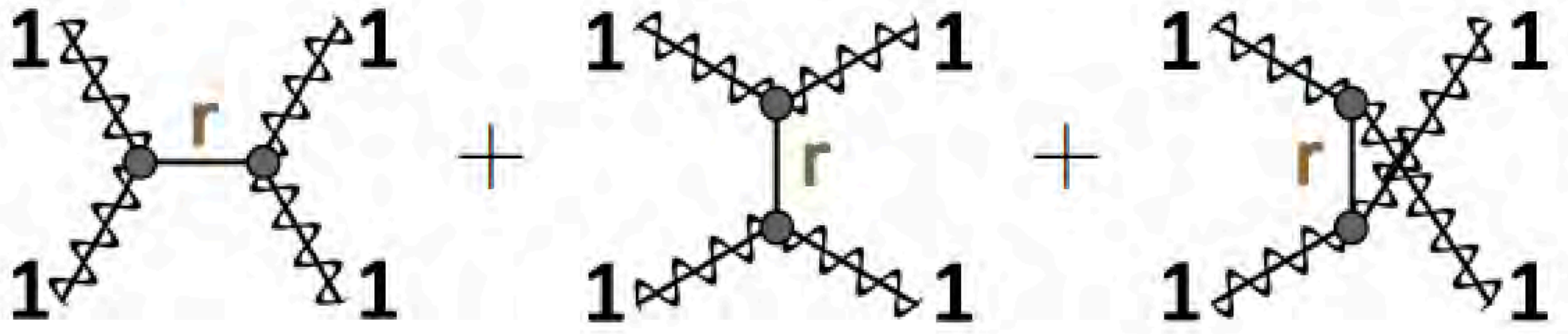


Lastly, there are 3 free parameters (κ,k,r_c). For illustration …

Lightest Spin-2 Mass:	$m_1 = 1$ TeV
Unitless Parameter:	$kr_c = 9.5$
4D Planck Mass:	$M_{\text{Pl}} = 2.435 \times 10^{15}$ TeV

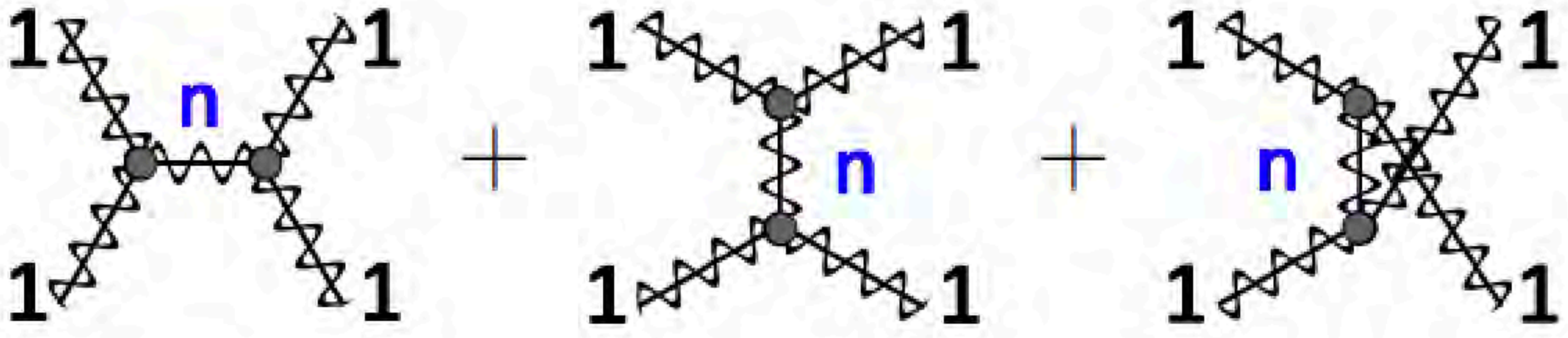
Individual Contributions to Helicity-0 RS1 Elastic KK Scattering

Scalar Exchange: $\mathcal{M}_r =$



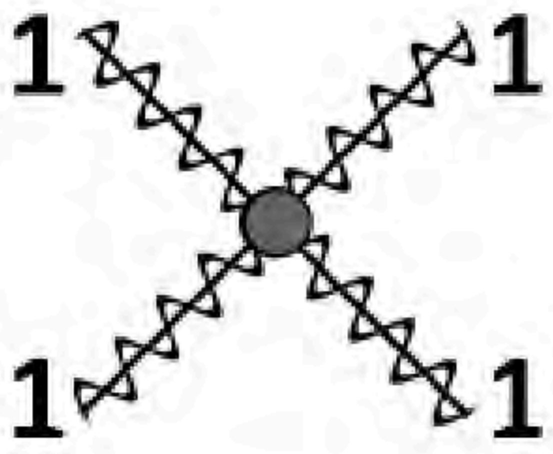
$\sim \mathcal{O}(s^3)$

KK Exchange: $\mathcal{M}_h(n) =$



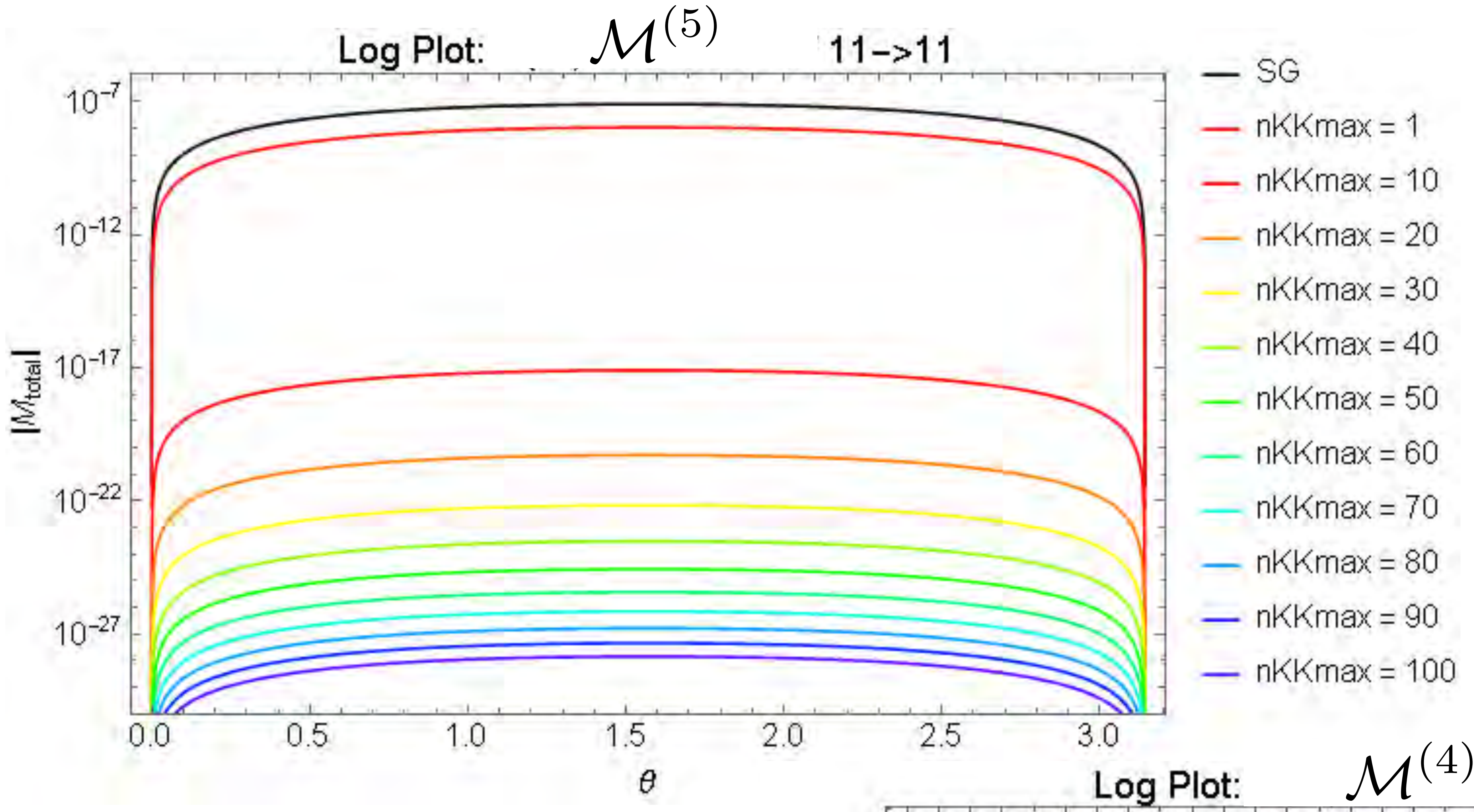
$\sim \mathcal{O}(s^5)$

Contact Interaction: $\mathcal{M}_{sg} =$



$\sim \mathcal{O}(s^5)$

Cancellations in the RS Model



$$\mathcal{M}(s,\theta) = \sum_{n=5,4,\ldots} \mathcal{M}^{(n)}(\theta) s^n$$

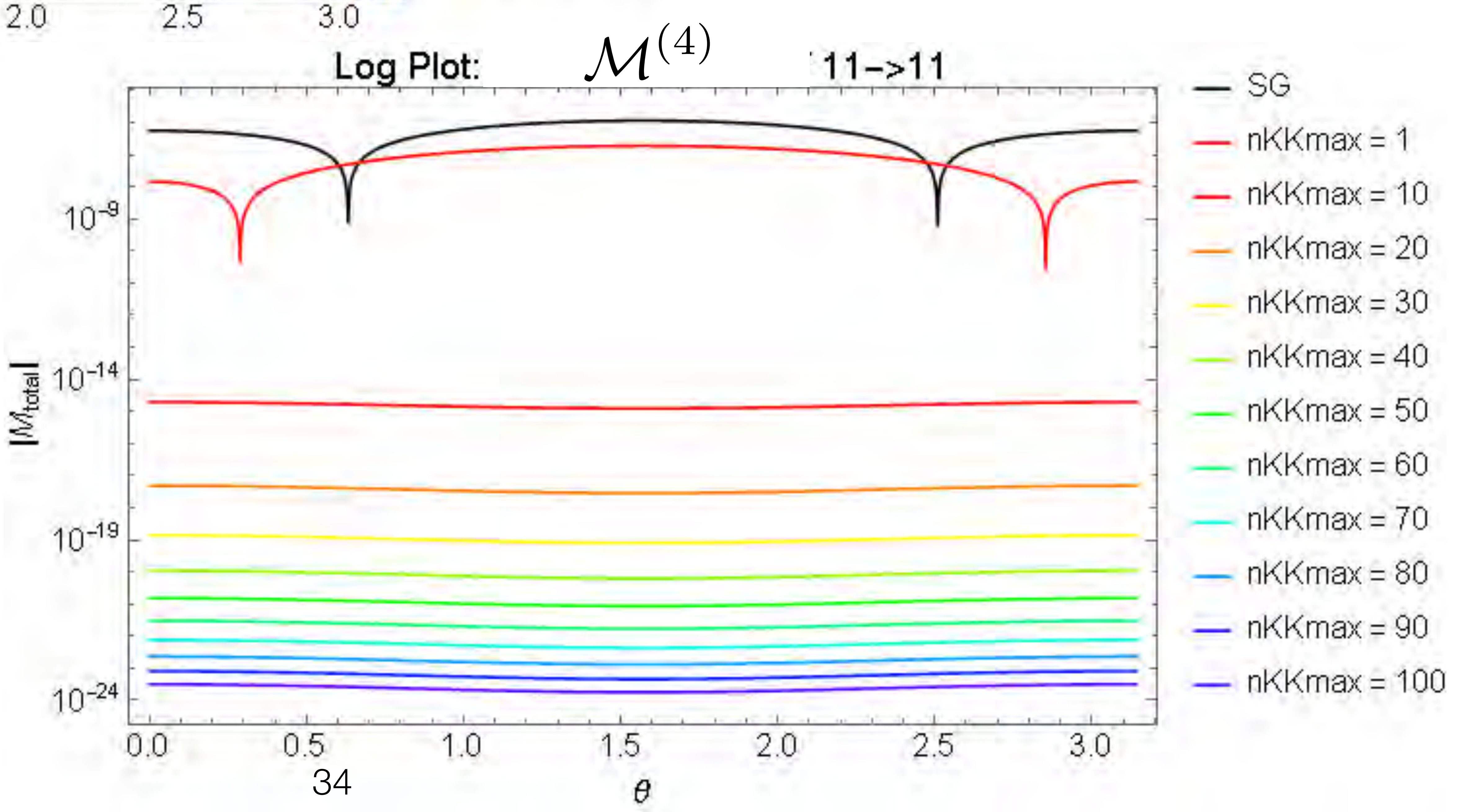
Summing over all intermediate KK states:

$$\mathcal{M}^{(5)} = \mathcal{M}^{(4)} = 0$$

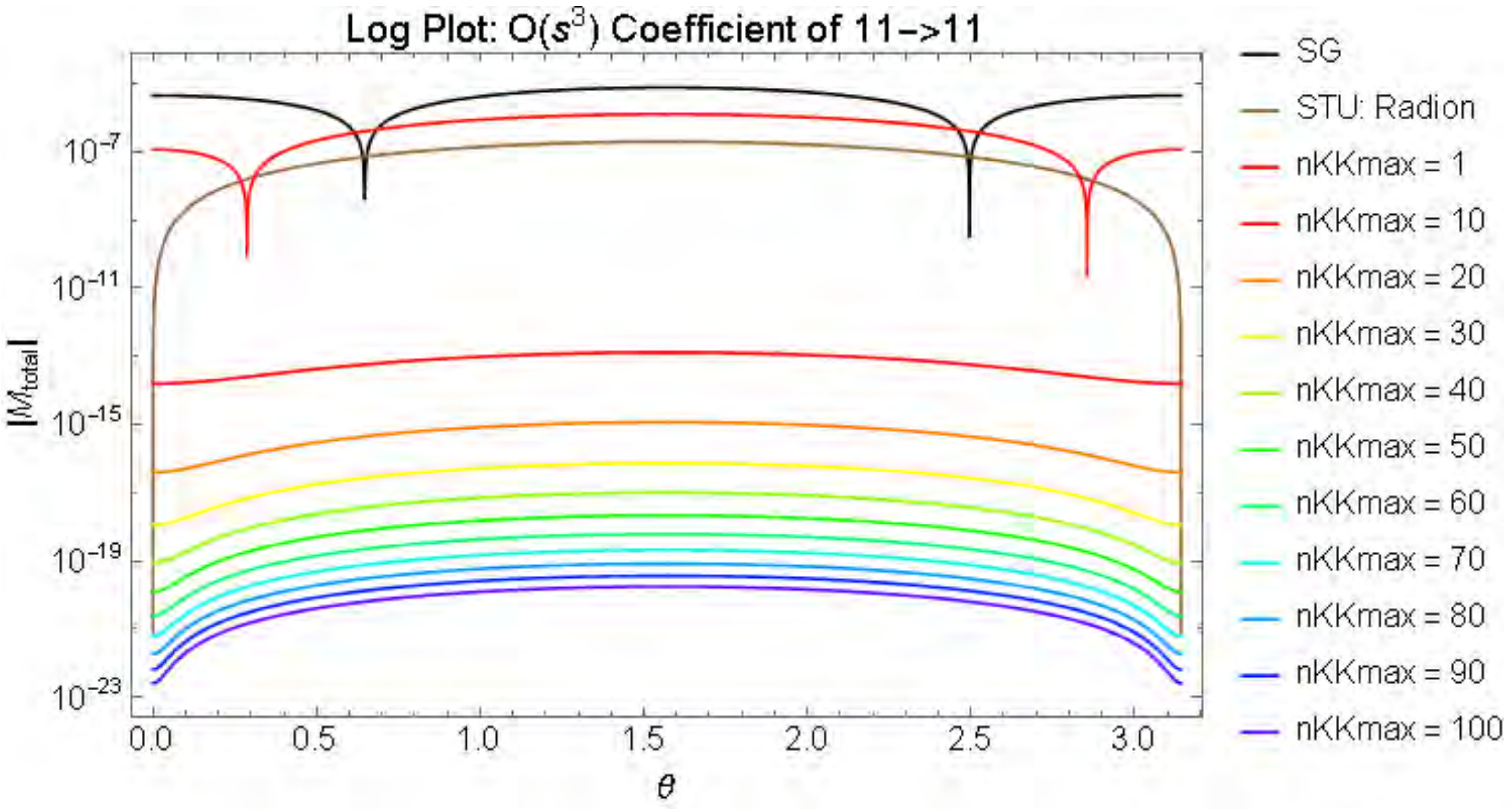
Cancellation enforced by sum-rules involving masses and couplings:

$$\sum_{j=0} a_{nnj}^2 = a_{nnnn} \ ,$$

$$\sum_{j=0} \mu_j^2 a_{nnj}^2 = \frac{4}{3}\mu_n^2 a_{nnnn} \ ,$$



Cancellations in the RS Model (cont'd)



Summing over all intermediate <u>KK *and* scalar</u> states:

$$\mathcal{M}^{(3)} = \mathcal{M}^{(2)} = 0$$

These sum-rules can be proved analytically…
residual amplitude O(s)!

Next: amplitudes for DM production

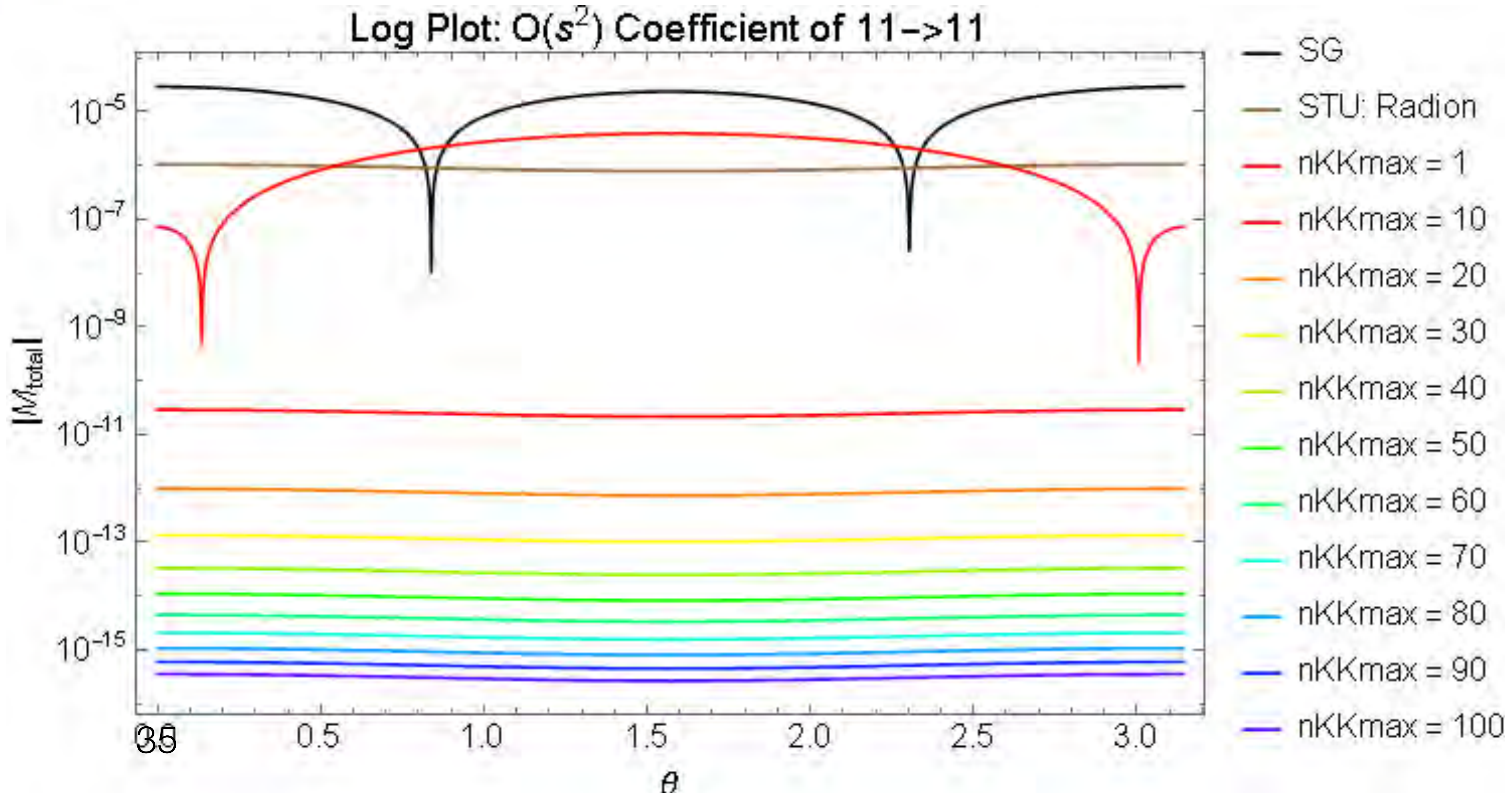
Cancellation enforced by sum-rules involving masses and couplings:

$$\sum_{j=0}^{+\infty} \mu_j^4 a_{nnj}^2 = \frac{4}{15}\mu_n^4(4\, a_{nnnn} - 3\, a_{nn0}^2) + \frac{36}{5}\sum_{i=0}^{+\infty} a_{n'n'(i)}^2 \ ,$$

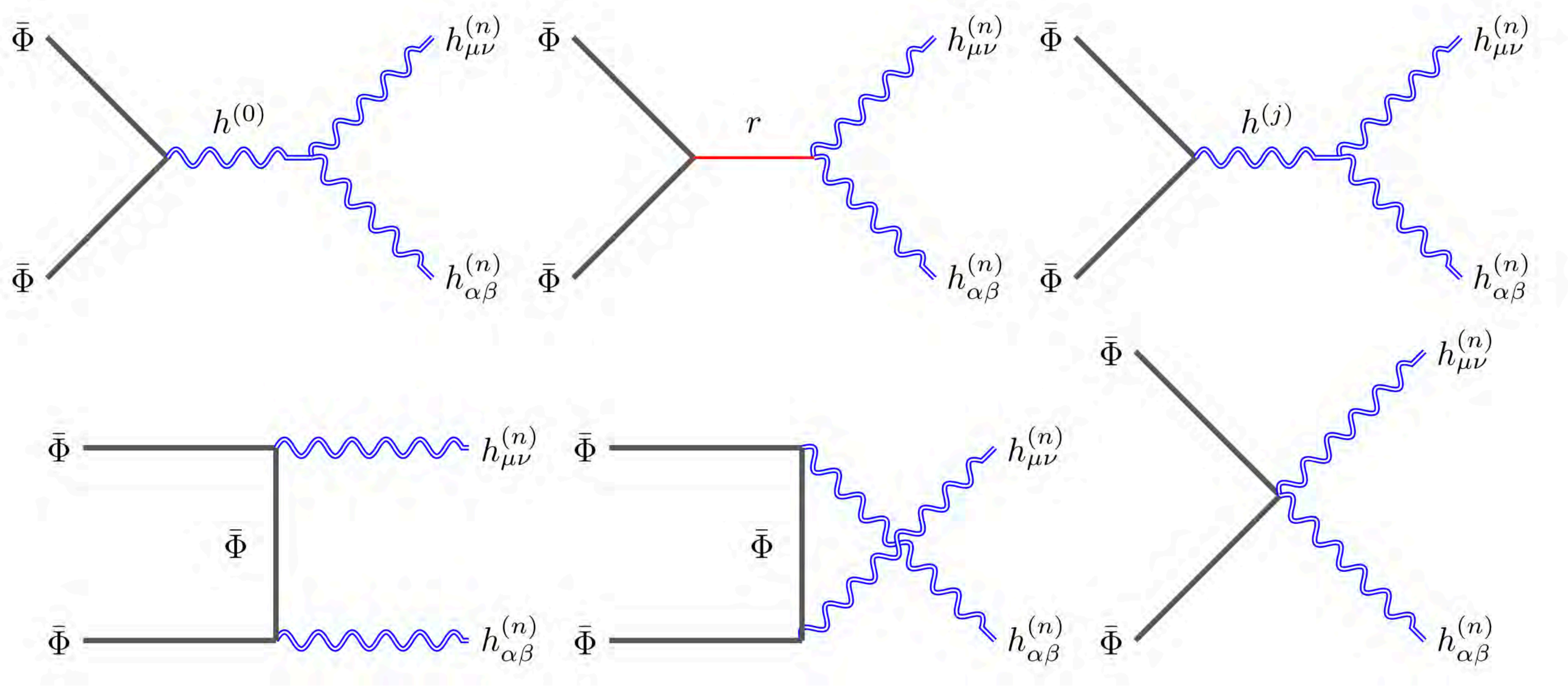
$$\sum_{j=0}^{+\infty} \mu_j^6 a_{nnj}^2 = -4\mu_n^6 a_{nn0}^2 + 9\sum_{i=0}^{+\infty}(4\mu_n^2 - \mu_{(i)}^2) a_{n'n'(i)}^2 \ ,$$

KK sum

scalar sum



Cancellations in DM Scattering to KK Gravitons



Sum rules ensure that $|\mathcal{M}| = \mathcal{O}\left(\frac{E^2}{M_{Pl}^2}\right)$

Symmetry, Gauge-fixing, & the Equivalence Theorem

Phys. Rev. D 106 (2022) 3, 035026 & 109 (2024) 7, 7

See also:

Gravitational Equivalence Theorem and Double-Copy for Kaluza-Klein Graviton Scattering Amplitudes

Yan-Feng Hang (Tsung-Dao Lee Inst., Shanghai and KLPAC, Shanghai and SKLPPC, Shanghai and Shanghai Jiaotong U.), Hong-Jian He (Tsung-Dao Lee Inst., Shanghai and KLPAC, Shanghai and SKLPPC, Shanghai and Shanghai Jiaotong U. and Tsinghua U., Beijing and Peking U., CHEP)
Jul 14, 2022

’t-Hooft-Feynman Gauge: Unphysical Goldstone Bosons

RS metric in conformal coordinates:

$$G_{MN} = e^{2A(z)} \begin{pmatrix} e^{-\kappa\hat{\varphi}/\sqrt{6}}(\eta_{\mu\nu} + \kappa\hat{h}_{\mu\nu}) & \frac{\kappa}{\sqrt{2}}\hat{A}_\mu \\ \frac{\kappa}{\sqrt{2}}\hat{A}_\mu & -\left(1 + \frac{\kappa}{\sqrt{6}}\hat{\varphi}\right)^2 \end{pmatrix}$$

$$A(z) = -\ln(kz) \qquad A'' - (A')^2 = 0$$

Bulk Einstein Equation

In unitary gauge: $A_\mu, \phi = 0$

Aside: N=2 Quantum Mechanical “SUSY”

Quantum Mechanical “SUSY” <u>relates</u> two eigenvalue systems

- Consider an eigenvalue system of the form $D^\dagger D\, f(x) = \lambda\, f(x)$.
  - Requires an inner produce to define $D^\dagger$, and therefore boundary conditions on the functions f(x).
- **Implies the system $DD^\dagger g(x)=\lambda g(x)$ has the same non-zero eigenvalues!**
  - $DD^\dagger(Df(x))= D(D^\dagger Df(x))=\lambda(Df(x))$ - hence $g(x)=Df(x)$ solves second system, so long as Df(x) doesn’t vanish!
- RS KK system hides TWO interleaved N=2 QM SUSY systems, which define the the tensor, vector, and scalar modes of the system and enforce their degeneracy!

’t-Hooft-Feynman Gauge: Mode Expansions and SUSY

Mode Expansions

$$\hat{h}_{\mu\nu}(x^\alpha, z) = \sum_{n=0}^{\infty} \hat{h}^{(n)}_{\mu\nu}(x^\alpha) f^{(n)}(z),$$

Vector Goldstone fields

$$\hat{A}_\mu(x^\alpha, z) = \sum_{n=1}^{\infty} \hat{A}^{(n)}_\mu(x^\alpha) g^{(n)}(z),$$

$$\hat{\varphi}(x^\alpha, z) = \hat{r}(x^\alpha) k^{(0)}(z) + \sum_{n=1}^{\infty} \hat{\pi}^{(n)}(x) k^{(n)}(z) \ ,$$

Scalar Goldstone fields

Einstein Equations relate sectors!

$$\overline{D}^\dagger \overline{D} - D D^\dagger = 2(A')^2 - 2A'' = 0 \ .$$

SUSY Relations in Vector-Graviton Sector:

$$D^\dagger D f^{(n)} = -(\partial_z + 3A')\partial_z f^{(n)} = m_n^2 f^{(n)},$$

$$D D^\dagger g^{(n)} = -\partial_z(\partial_z + 3A') g^{(n)} = m_n^2 g^{(n)},$$

$$\begin{cases} D f^{(n)} = m_n g^{(n)} \ , \\ D^\dagger g^{(n)} = m_n f^{(n)} \ , \end{cases}$$

$$D = \partial_z, \quad D^\dagger = -(\partial_z + 3A'),$$

SUSY Relations in Vector-Scalar Sector:

$$\overline{D}^\dagger \overline{D} g^{(n)} = -(\partial_z + 2A')(\partial_z + A') g^{(n)} = m_n^2 g^{(n)} \ ,$$

$$\overline{D}\,\overline{D}^\dagger k^{(n)} = -(\partial_z + A')(\partial_z + 2A') k^{(n)} = m_n^2 k^{(n)},$$

$$\begin{cases} \overline{D} g^{(n)} = m_n k^{(n)} \\ \overline{D}^\dagger k^{(n)} = m_n g^{(n)} \ . \end{cases}$$

$$\overline{D} = \partial_z + A', \quad \overline{D}^\dagger = -(\partial_z + 2A').$$

SUSY implies modes have same mass!

Supersymmetry and sum rules in the Goldberger-Wise model #3
R. Sekhar Chivukula (UC, San Diego), Elizabeth H. Simmons (UC, San Diego), Xing Wang (UC, San Diego) (Jul 6, 2022)
Published in: *Phys.Rev.D* 106 (2022) 3, 035026 • e-Print: 2207.02887 [hep-ph]

Supersymmetry in 5d gravity
C.S. Lim (Kobe U.), Tomoaki Nagasawa (Anan Coll. Tech.), Satoshi Ohya (Kobe U.), Kazuki Sakamoto (Kobe U.), Makoto Sakamoto (Kobe U.) (Oct, 2007)
Published in: *Phys.Rev.D* 77 (2008) 045020 • e-Print: 0710.0170 [hep-th]

`t-Hooft-Feynman Gauge Quadratic Terms

`t-Hooft-Feynman Gauge Fixing:

$$\mathcal{L}_{\rm GF} = \sum_n F^{(n)\mu} F^{(n)}_\mu - F^{(n)}_5 F^{(n)}_5,$$

$$F^{(n)}_\mu = -\left(\partial^\nu h^{(n)}_{\mu\nu} - \frac{1}{2}\partial_\mu h^{(n)} + \frac{1}{\sqrt{2}} m_n A^{(n)}_\mu\right),$$

$$F^{(n)}_5 = -\left(\frac{1}{2} m_n h^{(n)} - \frac{1}{\sqrt{2}}\partial^\mu A^{(n)}_\mu + \sqrt{\frac{3}{2}} m_n \varphi^{(n)}\right)$$

SUSY insures modes have same masses

`t-Hooft-Feynman Gauge Propagators:

$$\mathcal{D}^{\mu\nu\rho\sigma}_h = \frac{1}{2}\left(\eta^{\mu\rho}\eta^{\nu\sigma} + \eta^{\mu\sigma}\eta^{\nu\rho} - \eta^{\mu\nu}\eta^{\rho\sigma}\right)(-\Box - m_n^2),$$

$$\mathcal{D}^{\mu\nu}_A = -\eta^{\mu\nu}(-\Box - m_n^2),$$

$$\mathcal{D}_\varphi = -\Box - m_n^2.$$

Quadratic Mode Lagrangian:

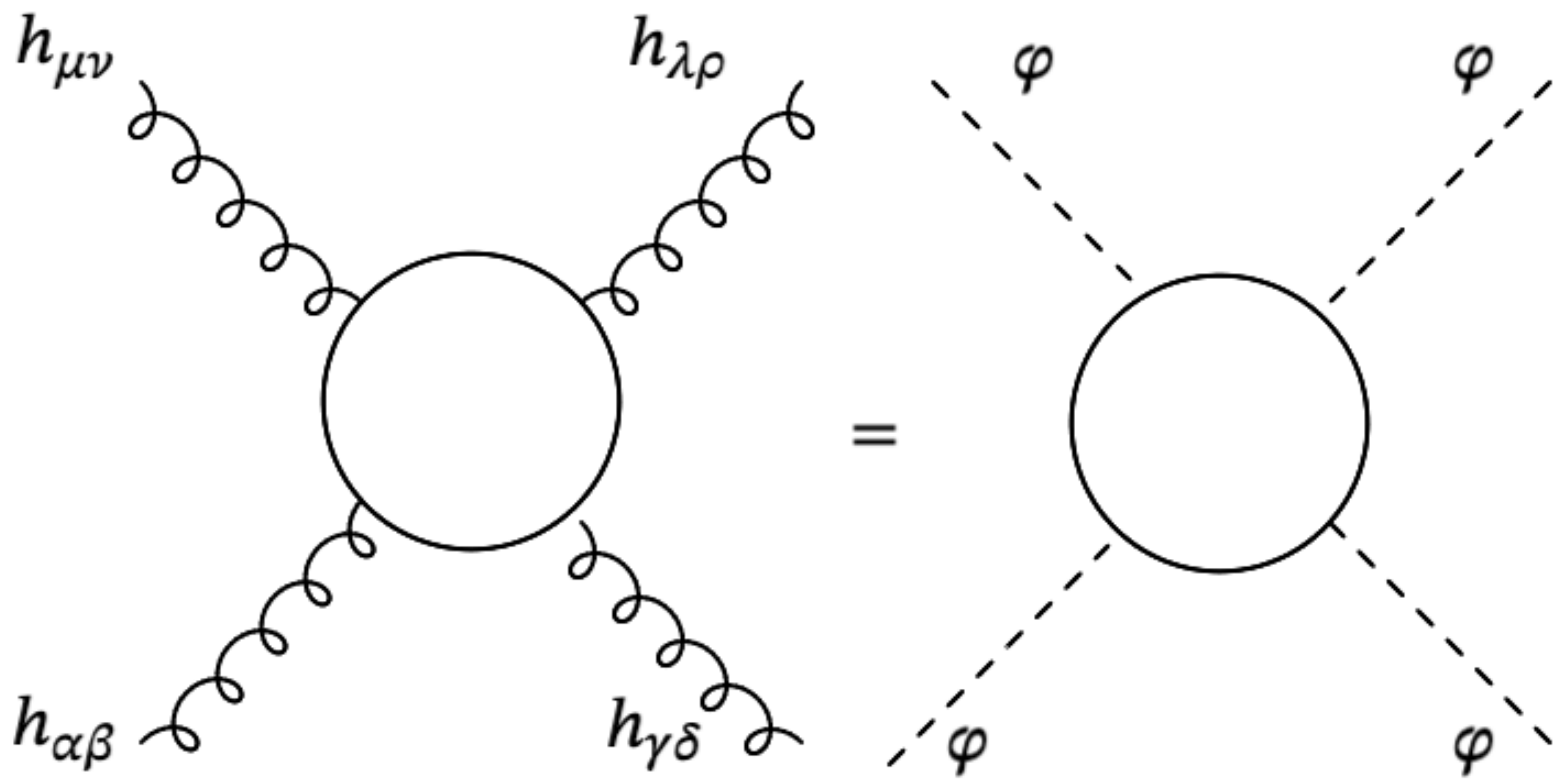
$$\mathcal{L}_2 = \sum_n \left(\frac{1}{2} h^{(n)}_{\mu\nu} \mathcal{D}^{\mu\nu\rho\sigma}_h h^{(n)}_{\rho\sigma} + \frac{1}{2} A^{(n)}_\mu \mathcal{D}^{\mu\nu}_A A^{(n)}_\nu + \frac{1}{2}\varphi^{(n)} D_\varphi \varphi^{(n)}\right)$$

No problematic "polarization projection" terms…

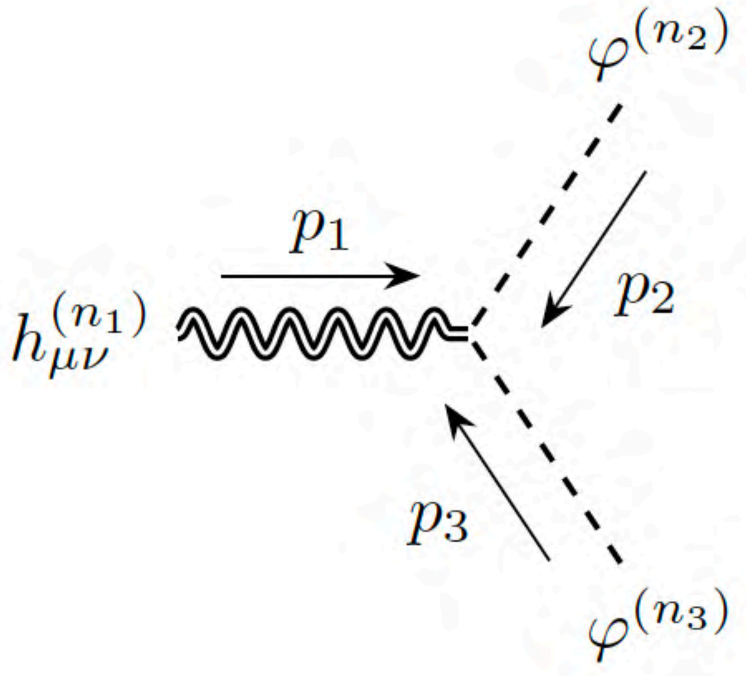
Goldstone Boson Equivalence Theorem

Equivalence Theorem:

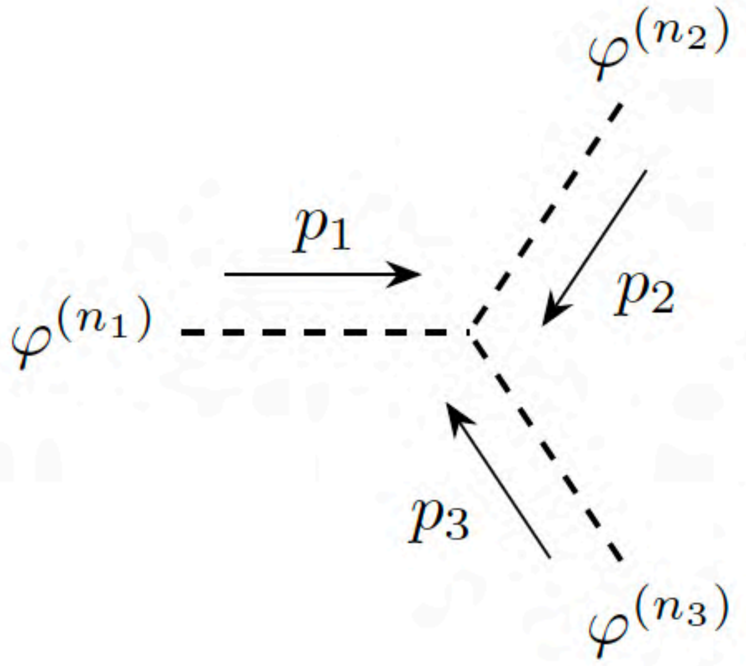
$$\mathcal{M}\left[h_L^{(n_1)} h_L^{(n_2)} \cdots\right] = \mathcal{M}\left[\varphi^{(n_1)} \varphi^{(n_2)} \cdots\right] + \mathcal{O}(s^0),$$



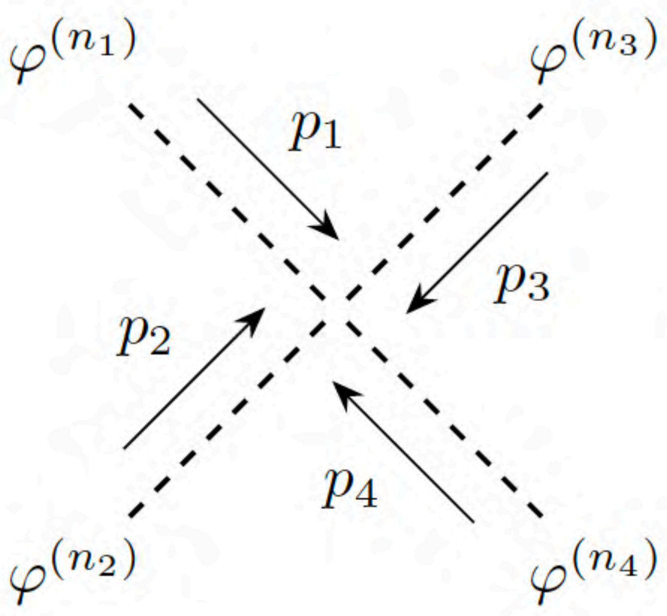
Goldstone Boson Interactions:



$$= \frac{i\kappa}{24}\langle f^{(n_1)} k^{(n_2)} k^{(n_3)}\rangle \left[4p_1^\mu p_1^\nu - 12 p_2^\mu p_2^\nu - 12 p_3^\mu p_3^\nu\right., \\ \left. -\eta^{\mu\nu}\left(2p_1^2 + 6p_2^2 + 6p_3^2\right)\right] + \mathcal{O}\left((p_i)^0\right)$$



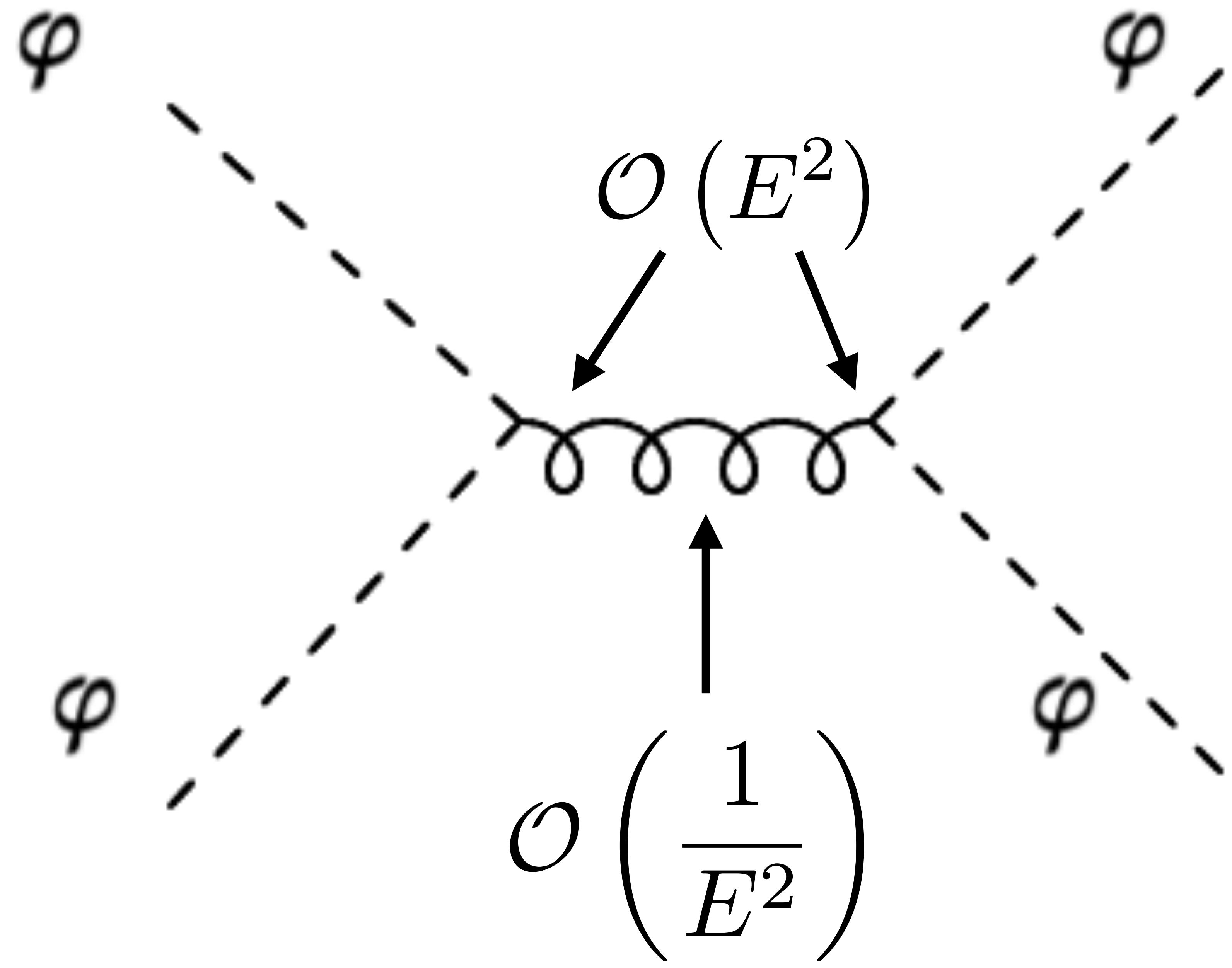
$$= -\frac{i\kappa}{\sqrt{6}}\langle k^{(n_1)} k^{(n_2)} k^{(n_3)}\rangle \left(p_1^2 + p_2^2 + p_3^2\right) + \mathcal{O}\left((p_i)^0\right),$$



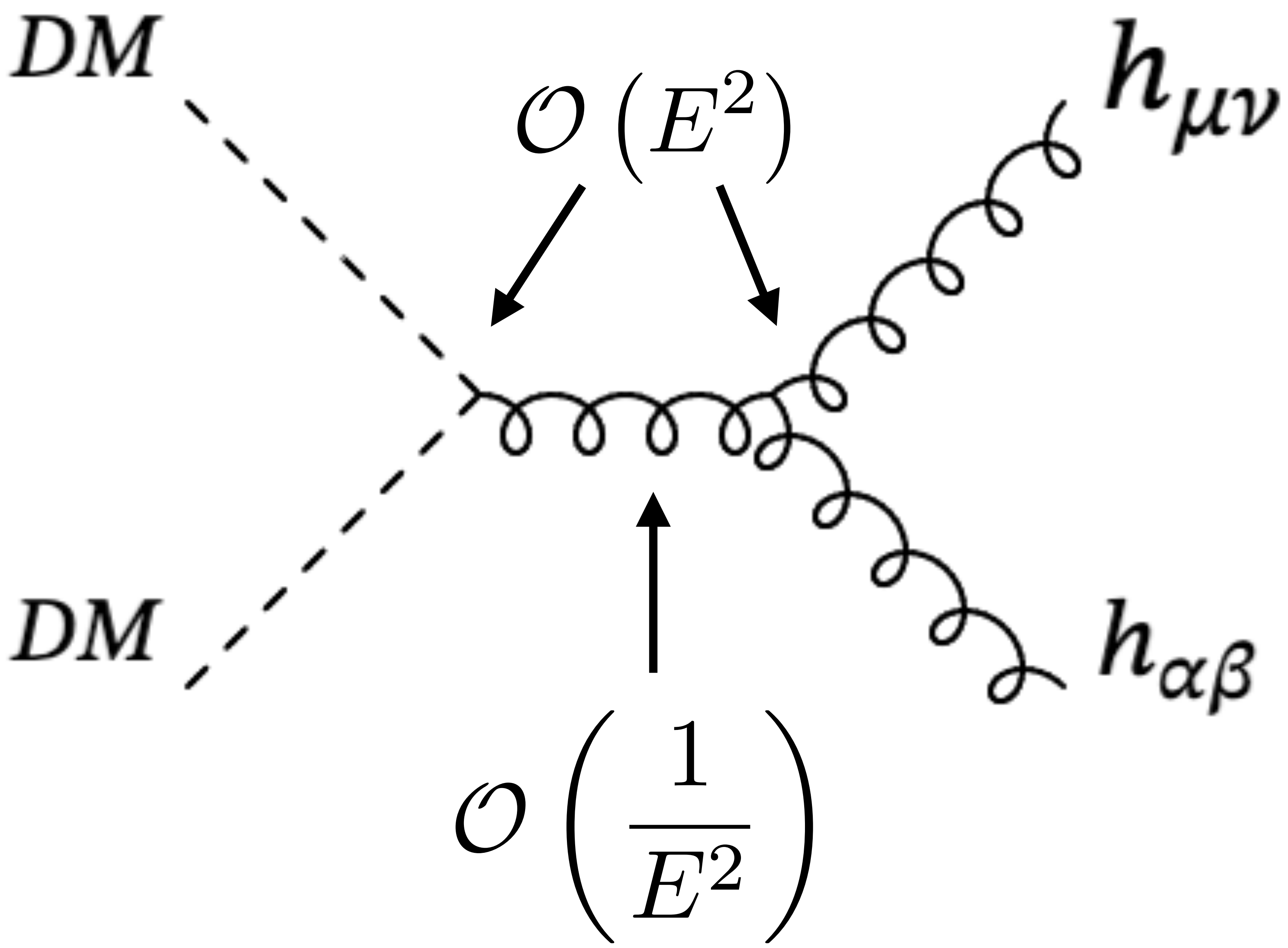
$$= \frac{i\kappa^2}{4}\langle k^{(n_1)} k^{(n_2)} k^{(n_3)} k^{(n_4)}\rangle \left(\sum_{i=1}^{4} p_i^2\right) + \mathcal{O}\left((p_i)^0\right).$$

’t-Hooft-Feynman Gauge Scattering Amplitudes: O(s)!

Elastic KK Scattering



DM KK Portal

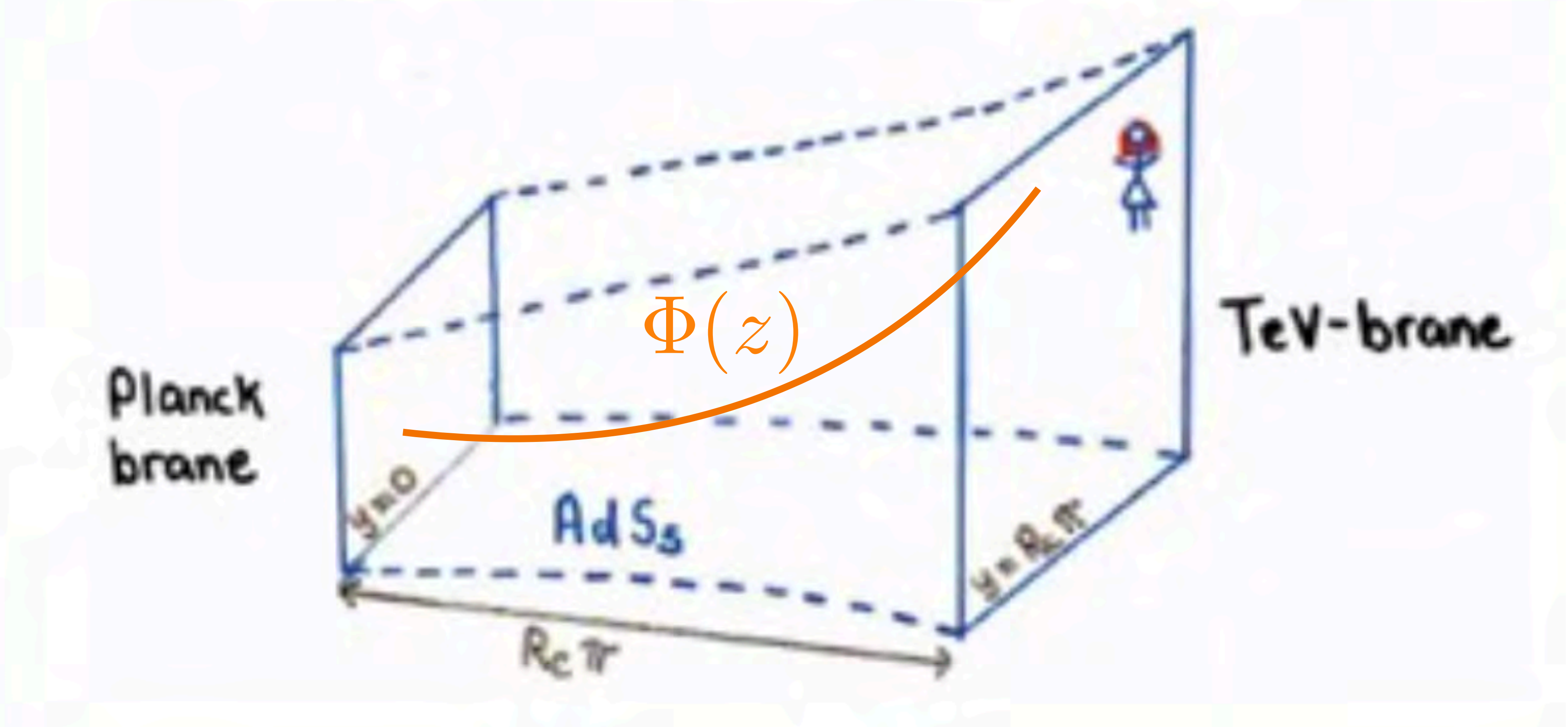


Since on-shell scattering amplitudes are gauge-invariant, the cancellations observed in unitary gauge must occur!

Generalization: Goldberger-Wise Stabilization

“Fix” values of Φ on branes, resulting in non-trivial background. Competition between kinetic and potential energy of scalar fixes size of extra dimension.

Gravitation-GW Scalar sectors mix, but retain SUSY for those modes “eaten” by the KK gravitons!



$$\mathcal{L}_{\Phi\Phi} = \sqrt{G}\left[\frac{1}{2}G^{MN}\partial_M\hat{\Phi}\partial_N\hat{\Phi}\right] ,$$

$$\mathcal{L}_{\rm pot} = -\frac{4}{\kappa^2}\left[\sqrt{G}V[\hat{\Phi}] + \sqrt{\overline{G}}V_1[\hat{\Phi}]\delta_1(z-z_1) + \sqrt{\overline{G}}V_2[\hat{\Phi}]\delta_1(z-z_2)\right]$$

$$\hat{\Phi}(x^\alpha, z) = \frac{1}{\kappa}(\phi_0(z) + \hat{\phi}(x^\alpha, z)).$$

Background EOM:

$$A'^2 - A'' = \frac{1}{12}(\phi_0')^2 ,$$
$$e^{2A}V = -6A'^2 + \frac{1}{8}(\phi_0')^2 ,$$
$$4e^{2A}\dot{V} = \phi_0'' + 3A'\phi_0' ,$$

$$\hat{\Psi}(x^\alpha, z) = \begin{pmatrix} \hat{\varphi}(x^\alpha, z) \\ \hat{\phi}(x^\alpha, z)\end{pmatrix}$$

$$\mathcal{L}_{h\text{-}\phi/\varphi} = -\hat{h}_{\mu\nu}\left[\sqrt{\frac{3}{2}}\eta^{\mu\nu}D^\dagger\left(\overline{\mathcal{D}}^\dagger\hat{\Psi}\right)_1\right],$$

$$\mathcal{L}_{A\text{-}\phi/\varphi} = \hat{A}_\mu\left[\sqrt{3}\,\partial^\mu\left(\overline{\mathcal{D}}^\dagger\hat{\Psi}\right)_1\right],$$

$$\mathcal{L}_{\phi/\varphi\text{-}\phi/\varphi} = \hat{\Psi}\left[-\frac{1}{2}\Box + \frac{1}{2}\overline{\mathcal{D}}\Lambda\overline{\mathcal{D}}^\dagger\right]\hat{\Psi} ,$$

$$\left(\overline{\mathcal{D}}^\dagger\hat{\Psi}\right)_1 = \left(-(\partial_z + 2A')\hat{\varphi} + \frac{1}{\sqrt{6}}\phi_0'\hat{\phi}\right)$$

$$\Lambda = \begin{pmatrix} 2 & \\ & -1\end{pmatrix}$$

$$\overline{\mathcal{D}} = \begin{pmatrix} \partial_z + A' & -\frac{1}{\sqrt{6}}\phi_0' \\ \frac{1}{\sqrt{6}}\phi_0' & -\frac{1}{\phi_0'}(\partial_z + 2A')\phi_0'\end{pmatrix}, \quad \overline{\mathcal{D}}^\dagger = \begin{pmatrix} -(\partial_z + 2A') & \frac{1}{\sqrt{6}}\phi_0' \\ -\frac{1}{\sqrt{6}}\phi_0' & \phi_0'(\partial_z + A')\frac{1}{\phi_0'}\end{pmatrix},$$

$$\overline{\mathcal{D}}^\dagger\overline{\mathcal{D}} = \begin{pmatrix} -(\partial_z + 2A')(\partial_z + A') + \frac{1}{6}\phi_0'^2 & \\ & -\phi_0'(\partial_z + A')\frac{1}{\phi_0'^2}(\partial_z + 2A')\phi_0' + \frac{1}{6}\phi_0'^2\end{pmatrix}$$

SUSY Relation follows <u>if EOM are satisfied</u>:

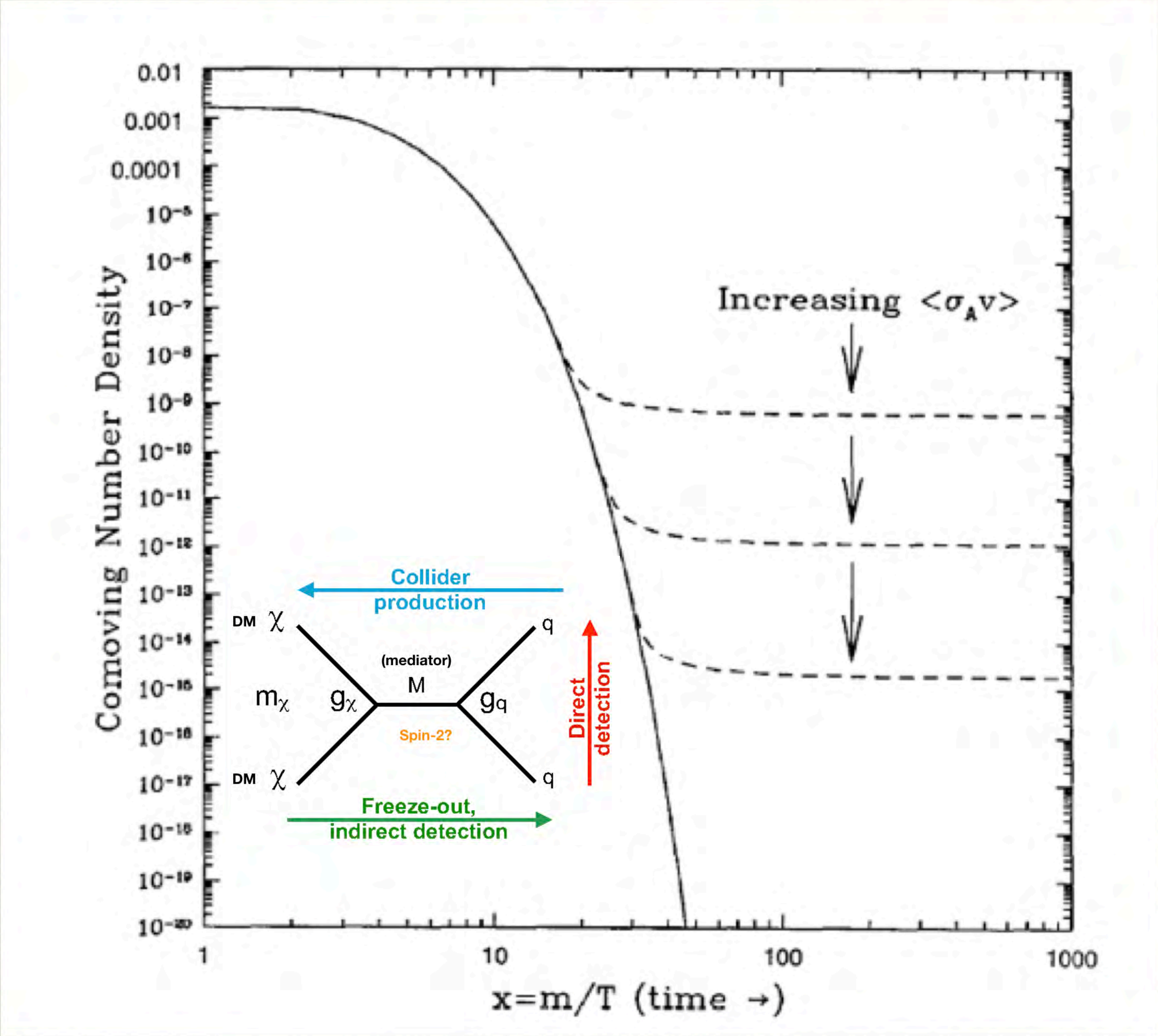
$$DD^\dagger = -\partial_z(\partial_z + 3A') = -(\partial_z + 2A')(\partial_z + A') + \frac{1}{6}\phi_0'^2 = H.$$

Equivalence theorem follows … scattering amplitudes O(s)

Part 3: KK Portal Dark Matter

Recall: Thermal Freeze-Out Cross Section

WIMPs and a Thermal Origin



Larger cross-section
⇒ later freeze-out
⇒ lower density

Correct DM density for:

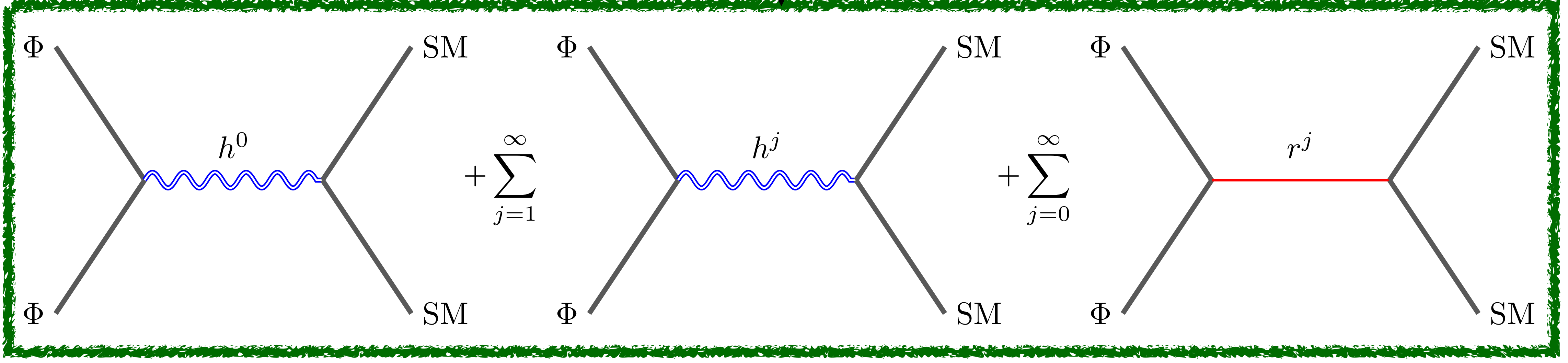
$$\langle \sigma v \rangle \simeq 3 \times 10^{-26}\ \mathrm{cm}^3/\mathrm{s}$$

$$\simeq \frac{1}{(20\ \mathrm{TeV})^2}$$

Schuster, 2019 FNAL DM Workshop

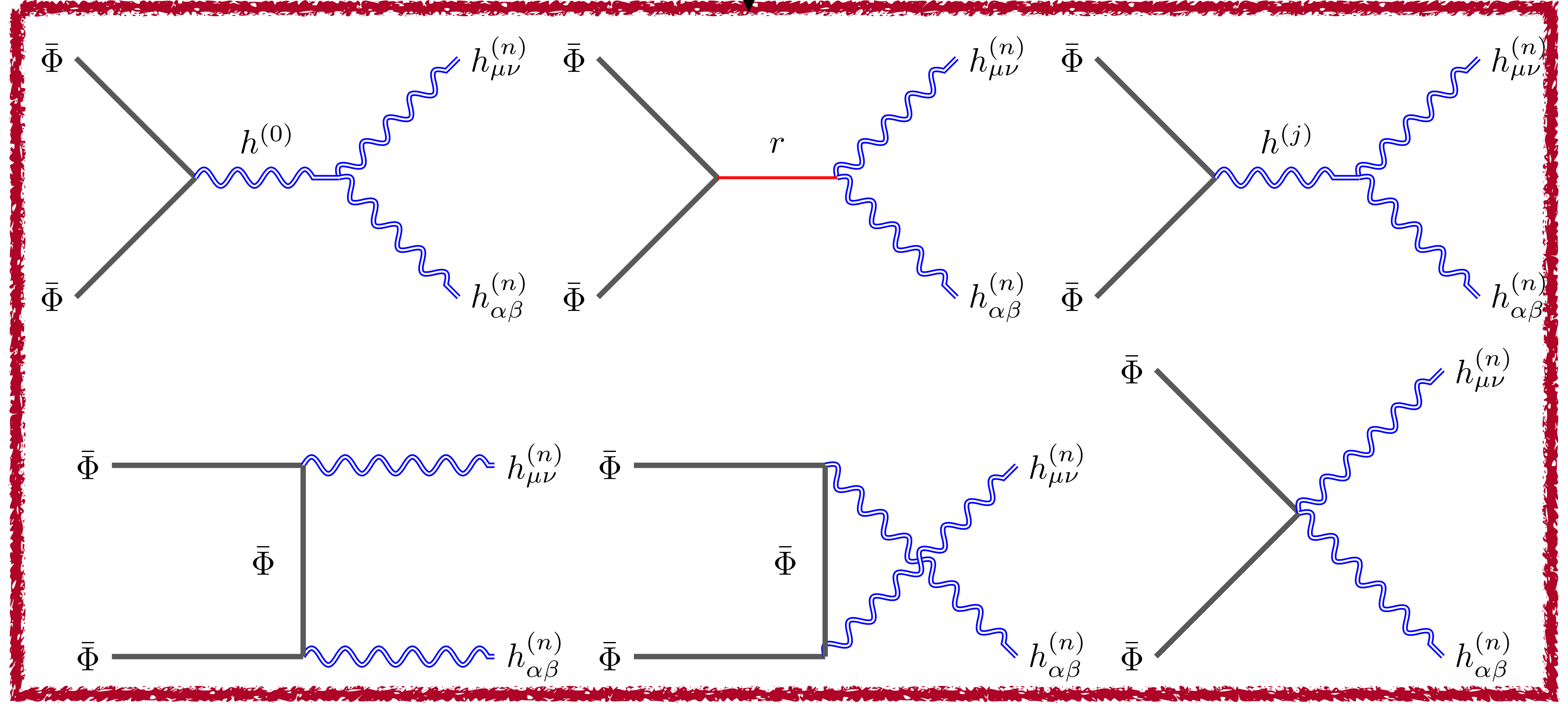
Summary of the issue on computing the relic density of KK portal DM

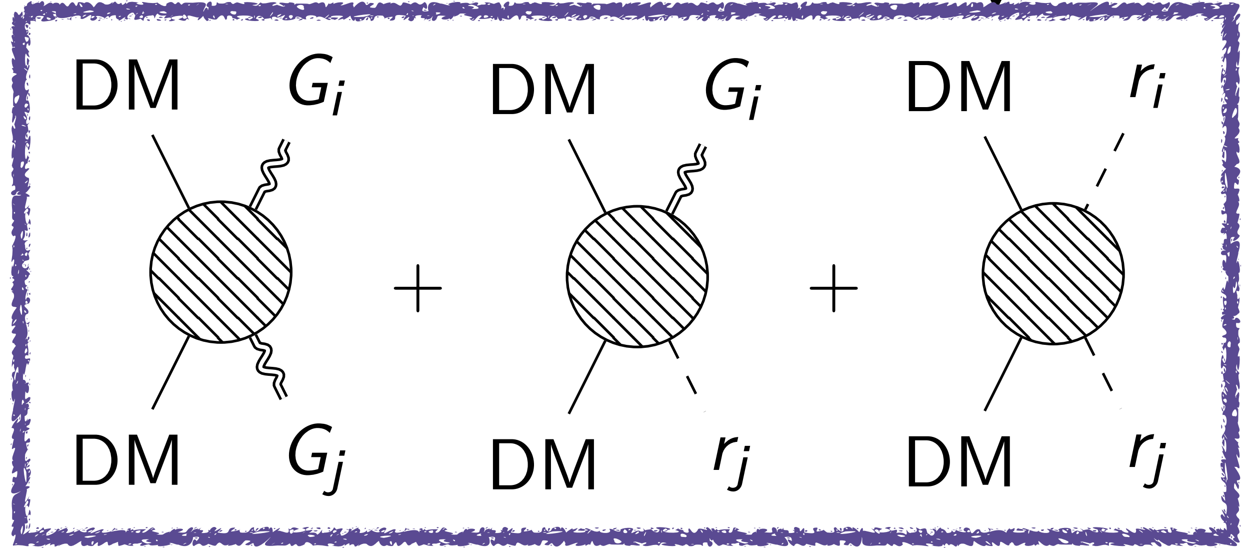
Three types of diagrams contribute to the calculation of relic densities



This set goes through resonant

Spin-2 and spin-0 modes, no anomalous energy growth





These two sets naively have divergences

due to helicity-0 (and 1) polarizations in external states

Problem and Cure

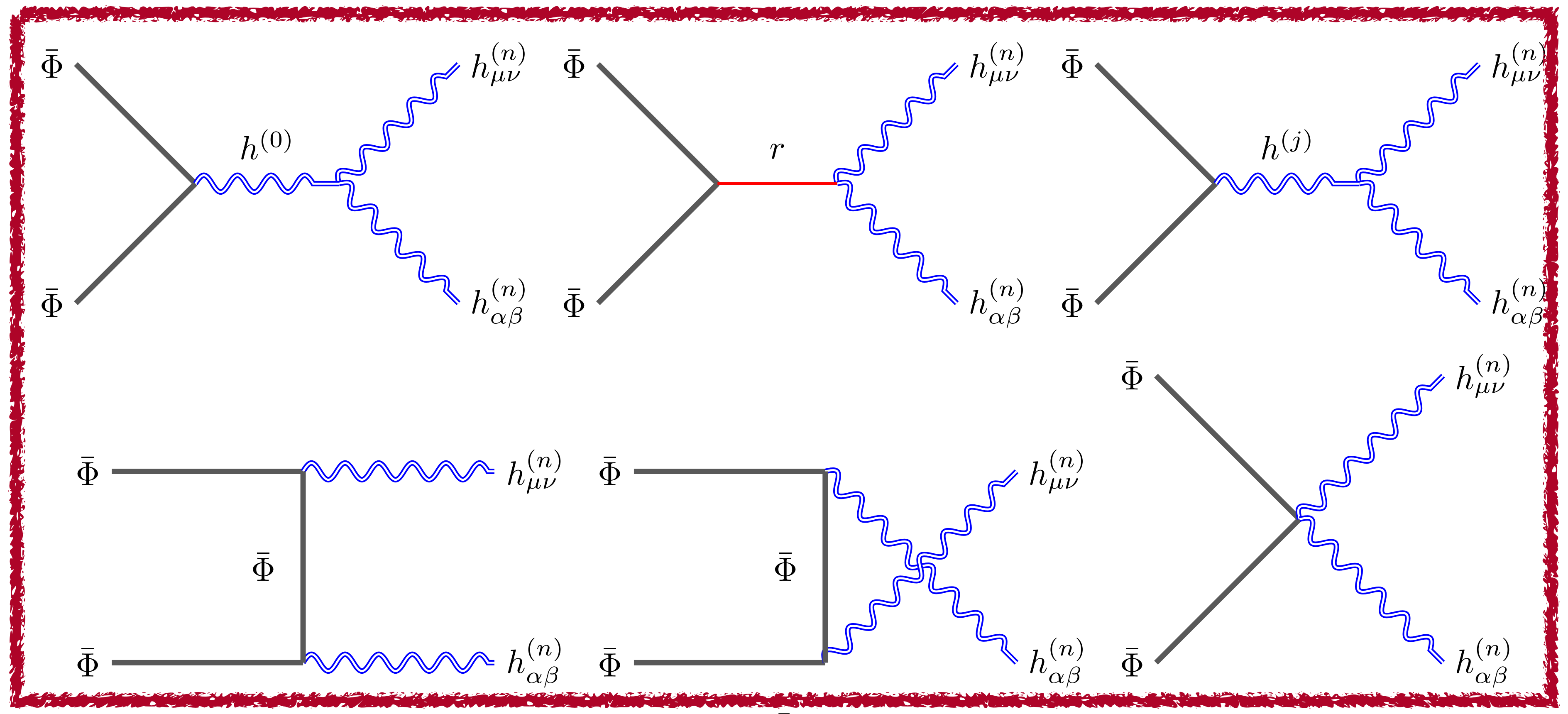


Diagram by diagram, these grows as $\mathcal{O}(s^3)$

$$\boxed{\widetilde{\mathcal{M}}^{(6)} = \frac{\kappa^2(1-\cos 2\theta)}{192 m_n^4}\left[\left(f^{(n)}(\bar{z})\right)^2 - \sum_{j=0}^{\infty} a_{nnj} f^{(j)}(\bar{z})\right]}$$

Sum all KK modes in all diagrams + GW scalar

Residual non-vanishing amplitude $\mathcal{O}(s)$

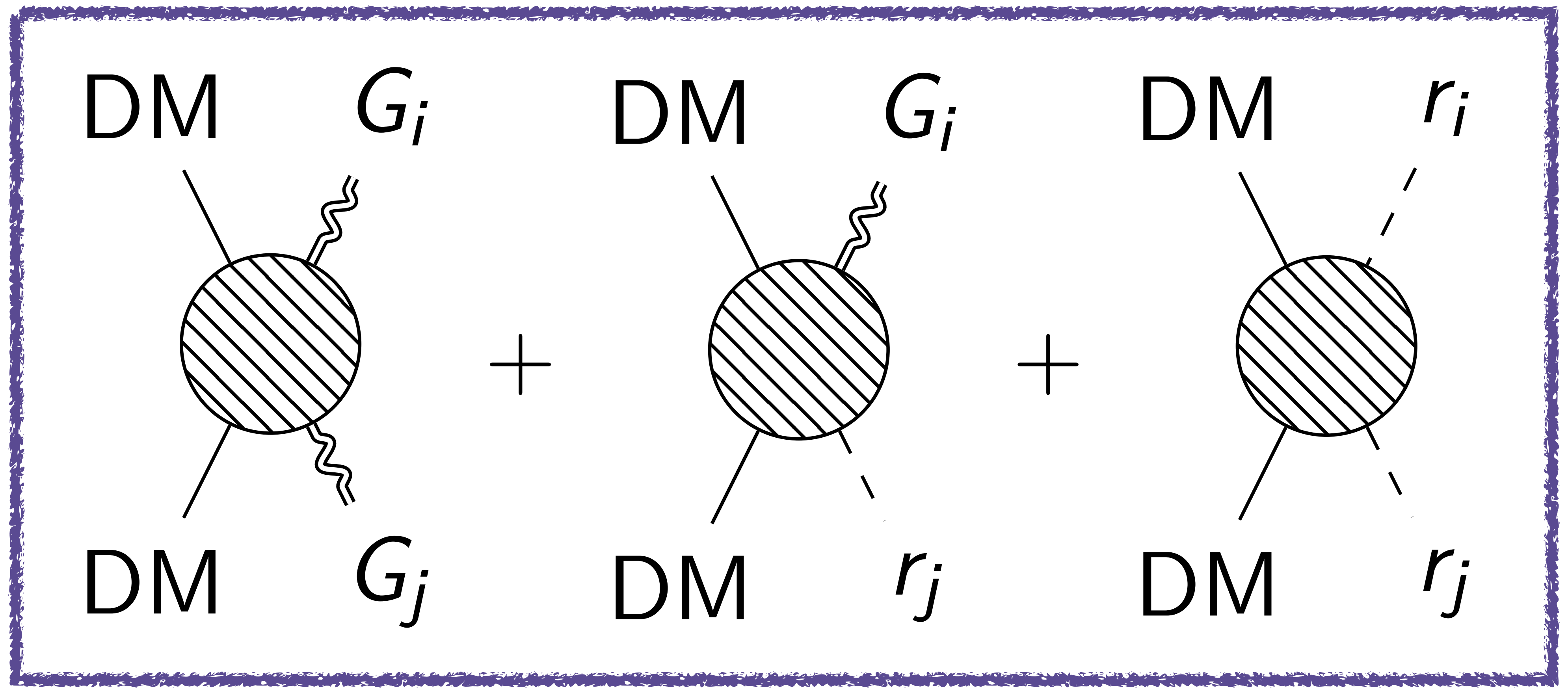


Diagram by diagram, these grow as $\mathcal{O}(s^2)$

Sum all KK modes in all diagrams + GW scalar

Residual non-vanishing amplitude $\mathcal{O}(s)$

- $SS \to G_i G_j$: $\mathcal{M} = \frac{i\kappa_i\kappa_j}{24}(1+3\cos 2\theta)\, s + \mathcal{O}(s^0)$
- $SS \to G_i r_j$: $\mathcal{M} = \frac{i\kappa_i\kappa_j^{(r)}}{24}(1+3\cos 2\theta)\, s + \mathcal{O}(s^0)$
- $SS \to r_i r_j$: $\mathcal{M} = -\frac{i\kappa_i^{(r)}\kappa_j^{(r)}}{24}(1+3\cos 2\theta)\, s + \mathcal{O}(s^0)$

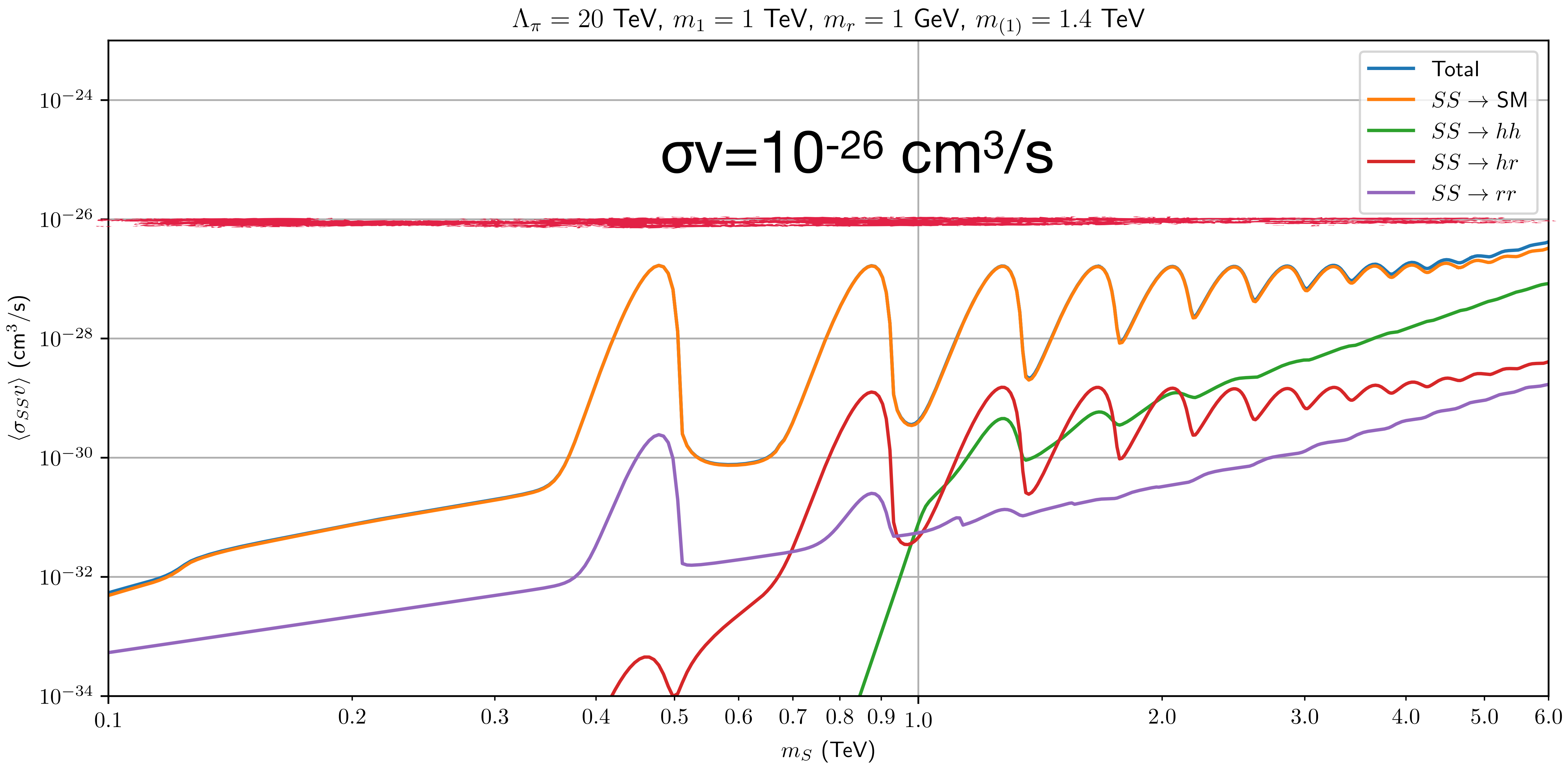
KK Portal DM Freeze-Out

Velocity Averaged cross sections

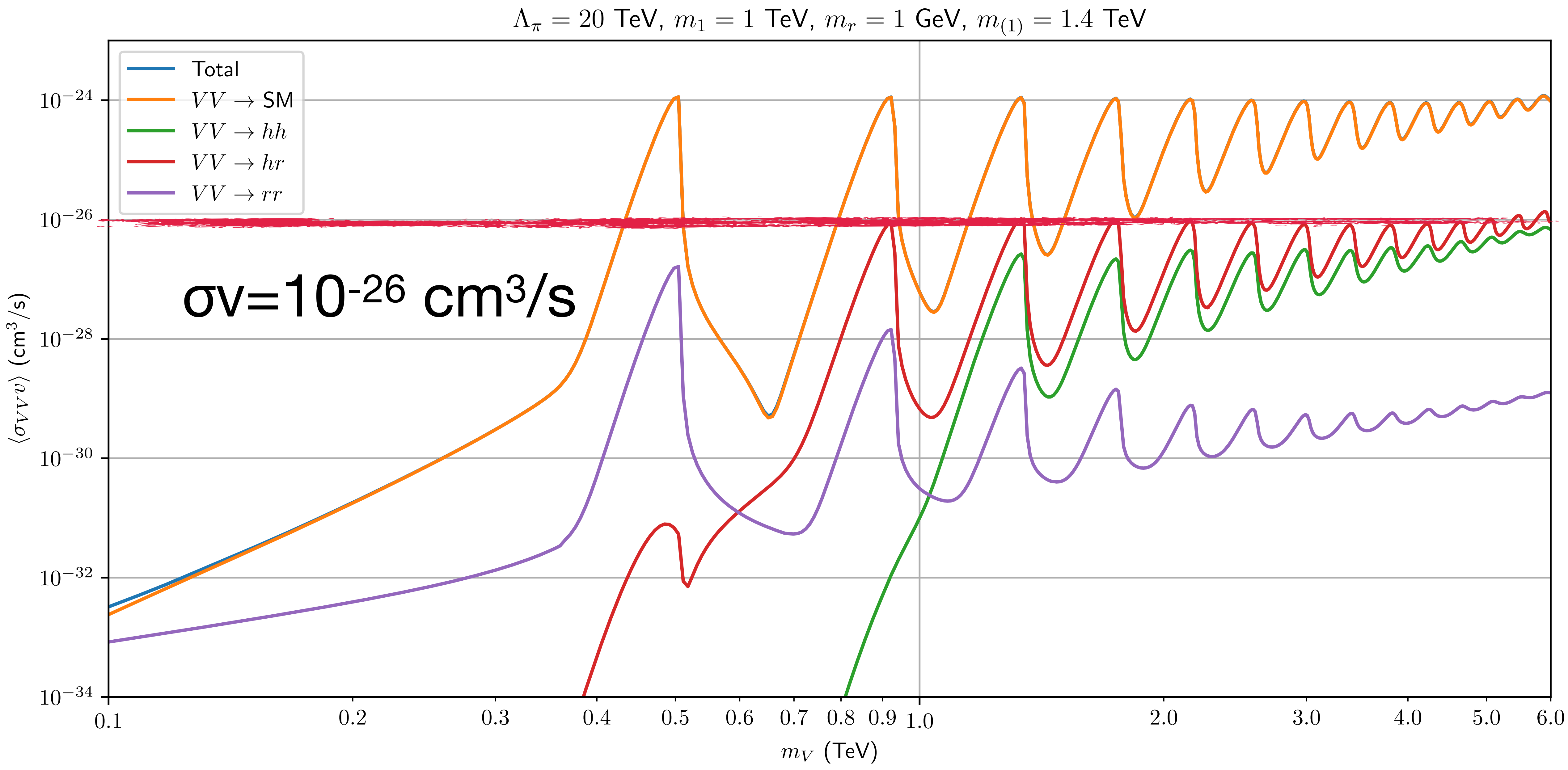
For freeze-out, we need velocity averaged cross-sections

$$\langle \sigma_{\Phi\Phi\to ff}\, v_{rel} \rangle = \frac{2\pi^2 T \int_{4m_\Phi^2}^{\infty} ds \sqrt{s}\,(s-4m_\Phi^2)\, K_1\left(\frac{\sqrt{s}}{T}\right) \sigma_{\Phi\Phi\to ff}(s)}{\left(4\pi m_\Phi^2 T K_2\left(\frac{m_\Phi}{T}\right)\right)^2}.$$

Scalar dark matter



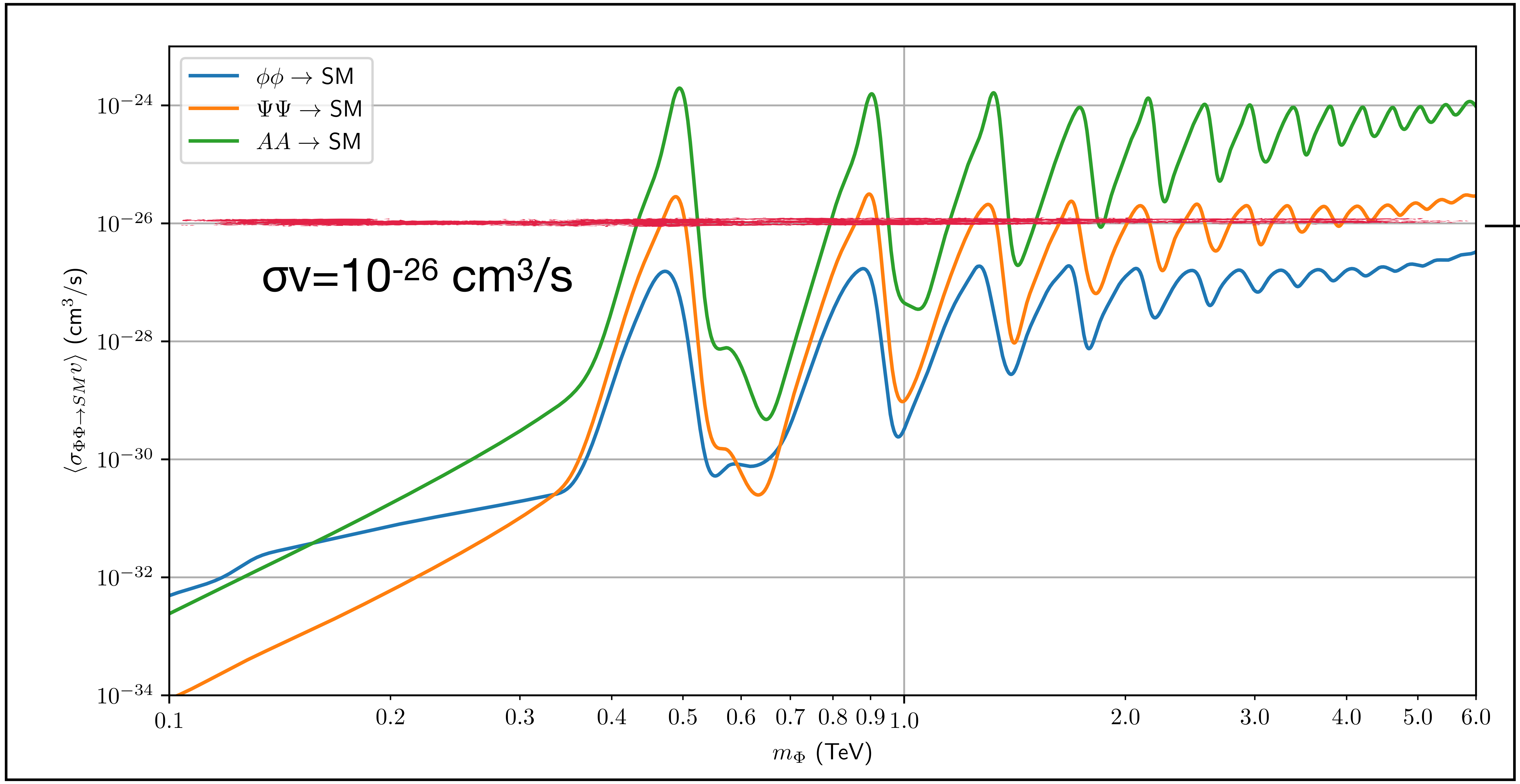
U(1) Vector dark matter



Standard Model Contribution dominates over annihilation of KK states - Counting!

Velocity Averaged cross sections

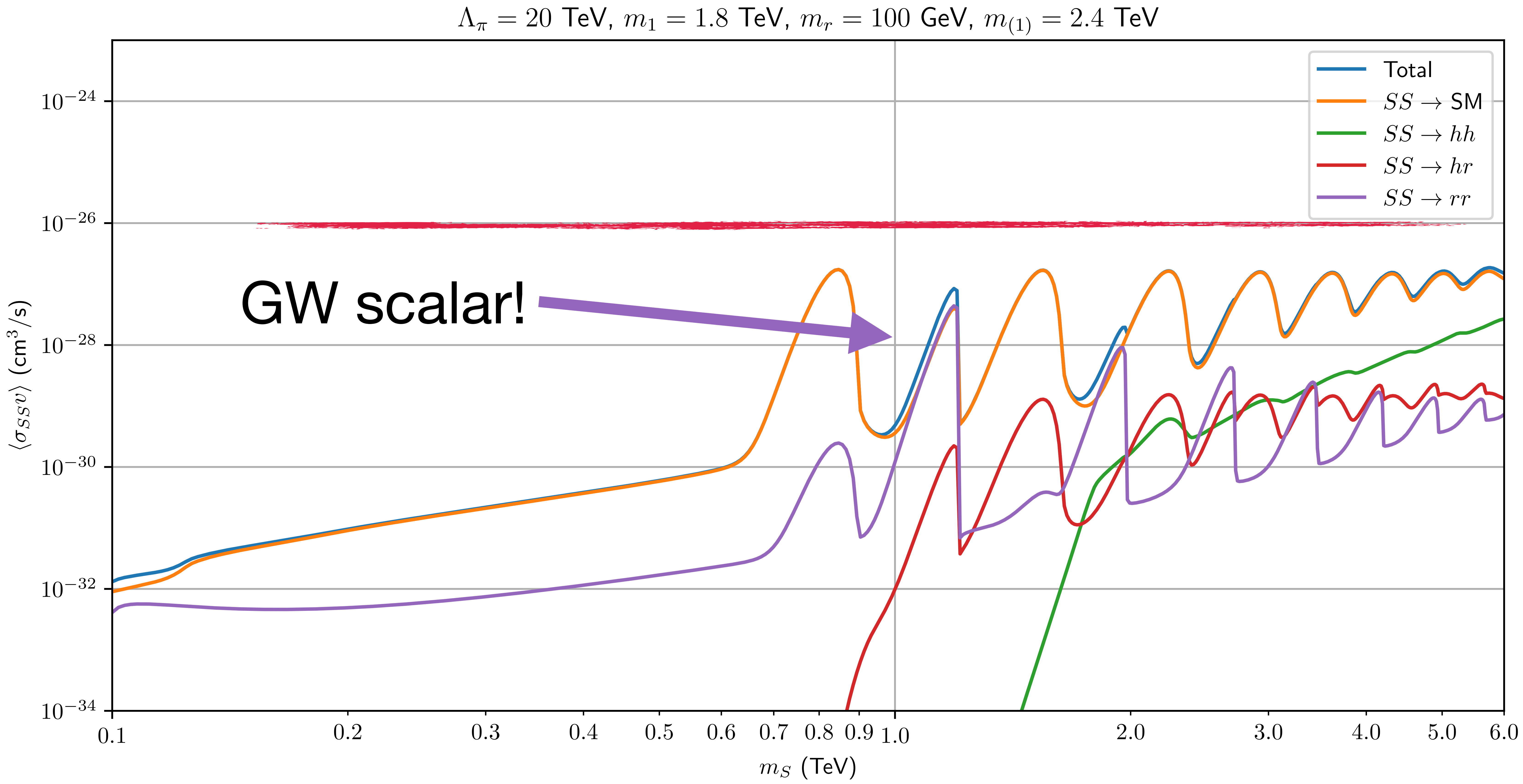
$\Lambda_\pi = 20$ TeV, $m_1 = 1$ TeV, $m_r = 1$ GeV, $m_{(1)} = 1.4$ TeV



Cross-section needed to saturate the relic density

- Vector dark matter can easily saturate the relic density due to s-wave annihilation
- Scalars are suppressed by a d-wave annihilation, Fermions by p-wave

(Work in progress: enhanced cross section in the case of a massive radion…)



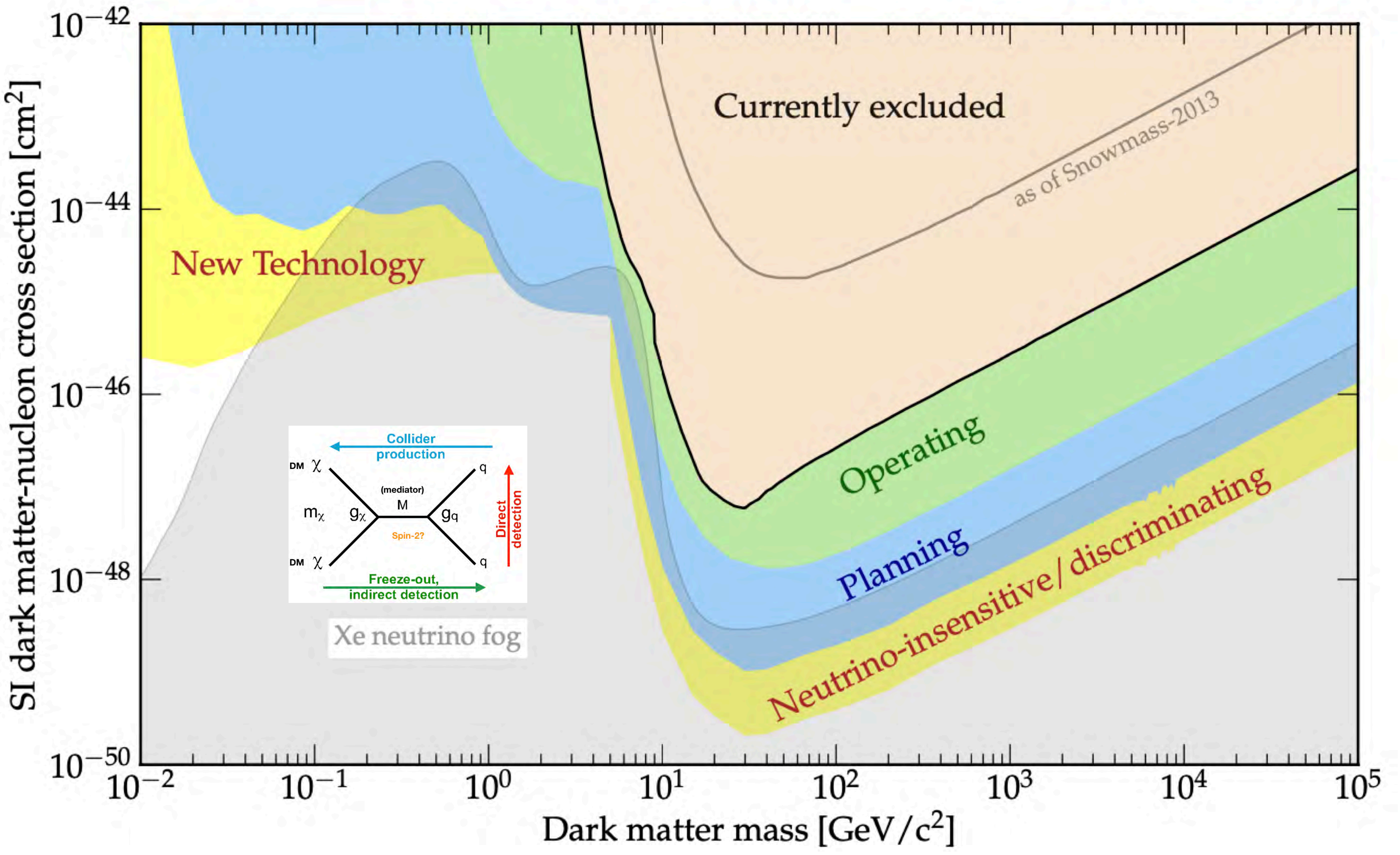
→ Cross-section needed to saturate the relic density

- Enhanced scalar DM cross section with massive (100 GeV) radion.
- New resonant contributions (purple) with Goldberger-Wise scalar exchange.
  - These couplings are proportional to radion mass … exploring higher masses!

KK Portal Dark Matter Detection

Dark Matter Direct Detection Constraints

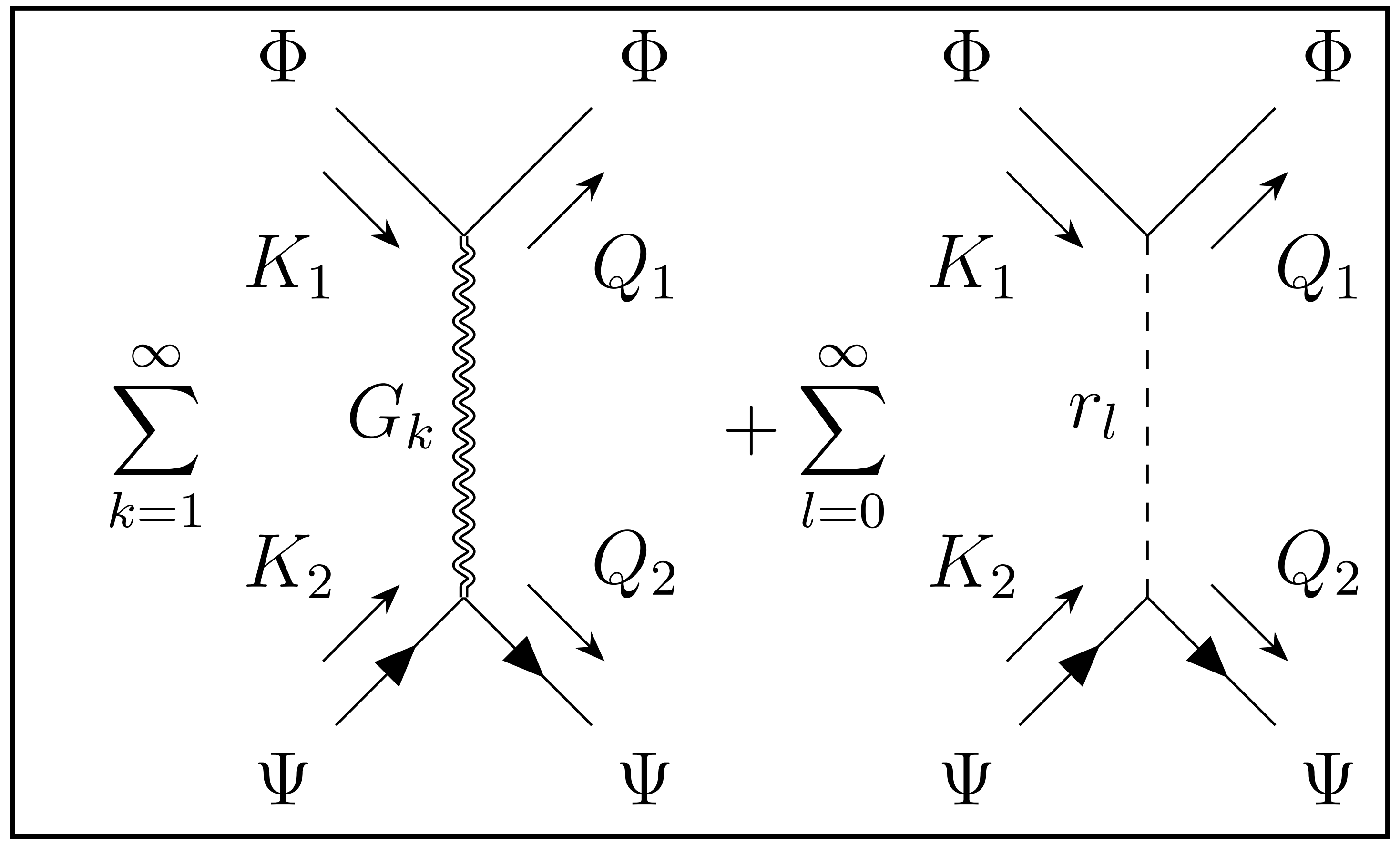
Limits on DM interaction strength with matter: weaker than weak!

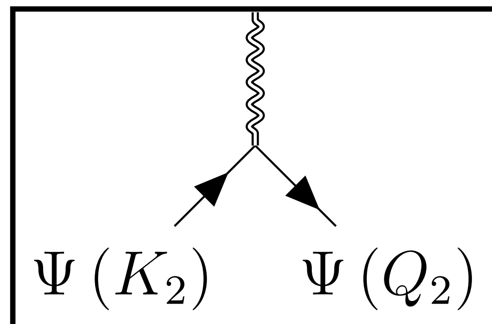


Jocelyn Monroe, ICHEP 2024

Direct Detection (coherent coupling to energy-momentum tensor)

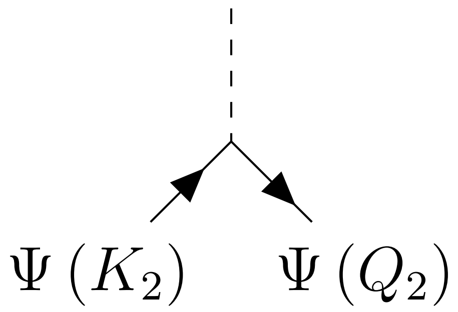
Dark Matter scattering of Nucleons





$$\propto -\frac{1}{4}\bar{u}_\Psi(Q_2)\left[\gamma^\mu\left(K_2^\nu+Q_2^\nu\right)+\gamma^\nu\left(K_2^\mu+Q_2^\mu\right)-\right.$$

$$\left.-2\eta^{\mu\nu}\left(\not{K}_2+\not{Q}_2-2m_\Psi\right)\right]u_\Psi(K_2)=V_2^{\mu\nu},$$



$$\propto -\sqrt{\frac{2}{3}}\bar{u}_\Psi(Q_2)\left[\frac{3}{4}\left(\not{Q}_2+\not{K}_2\right)-2m_\Psi\right]u_\Psi(K_2)=V_0,$$

$$V_2^{\mu\nu}=\left(V_2^{\mu\nu}-\frac{1}{4}\eta^{\mu\nu}V_2\right)+\frac{1}{2}\eta^{\mu\nu}V_2=\tilde{T}_\Psi^{\mu\nu}+\frac{1}{4}\eta^{\mu\nu}T_\Psi$$

Twist-2 operator

Twist-0 operator

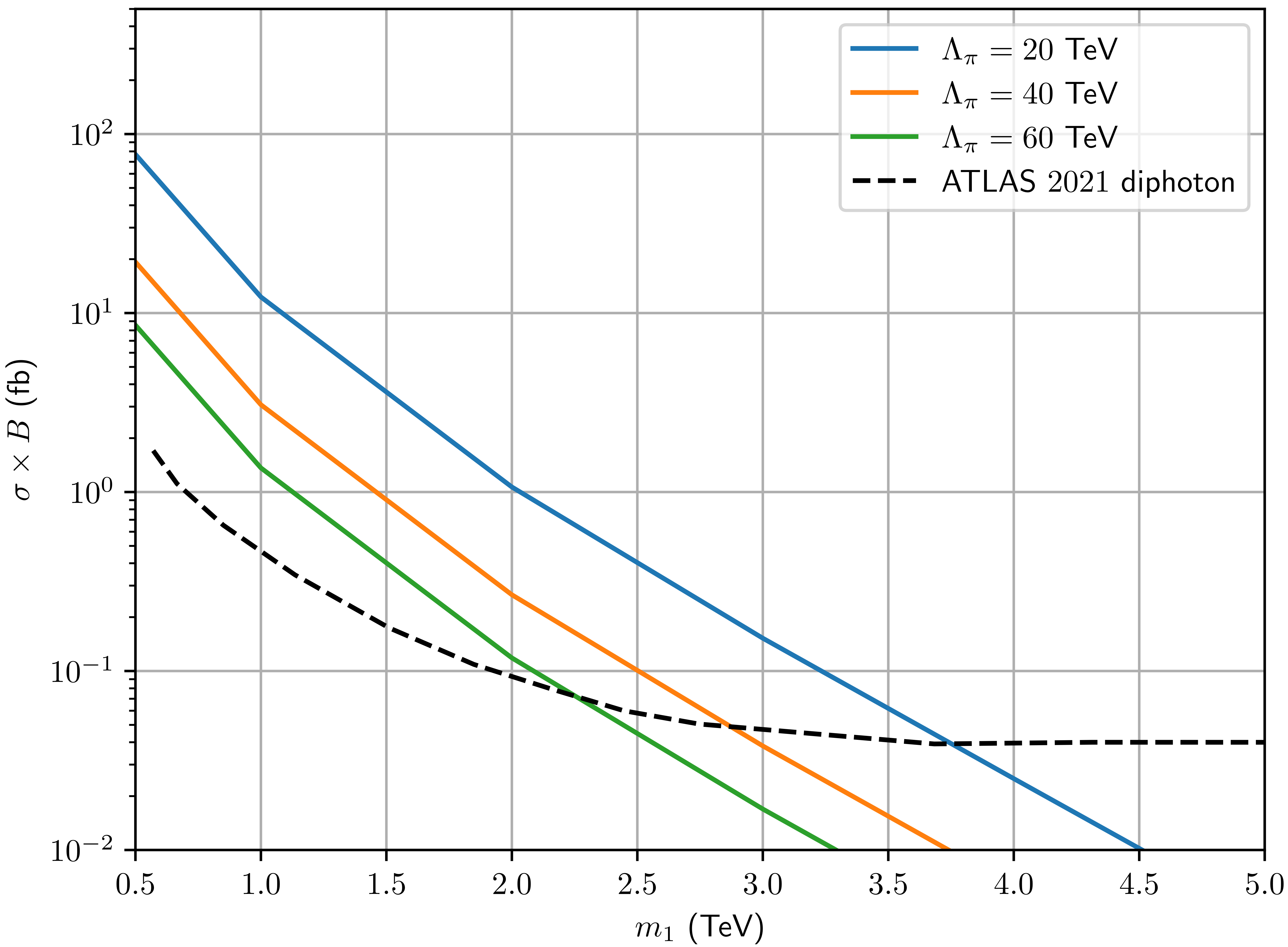
Squared Amplitude

$$\overline{\sum|\mathcal{M}|^2}=\frac{m_\Phi^2}{9}\sum_{\Psi=\{u,d,s\}}\left(Zm_p^2\bar{\mathcal{M}}_\Psi^p+(A-Z)\,m_n^2\bar{\mathcal{M}}_\Psi^n\right)^2$$

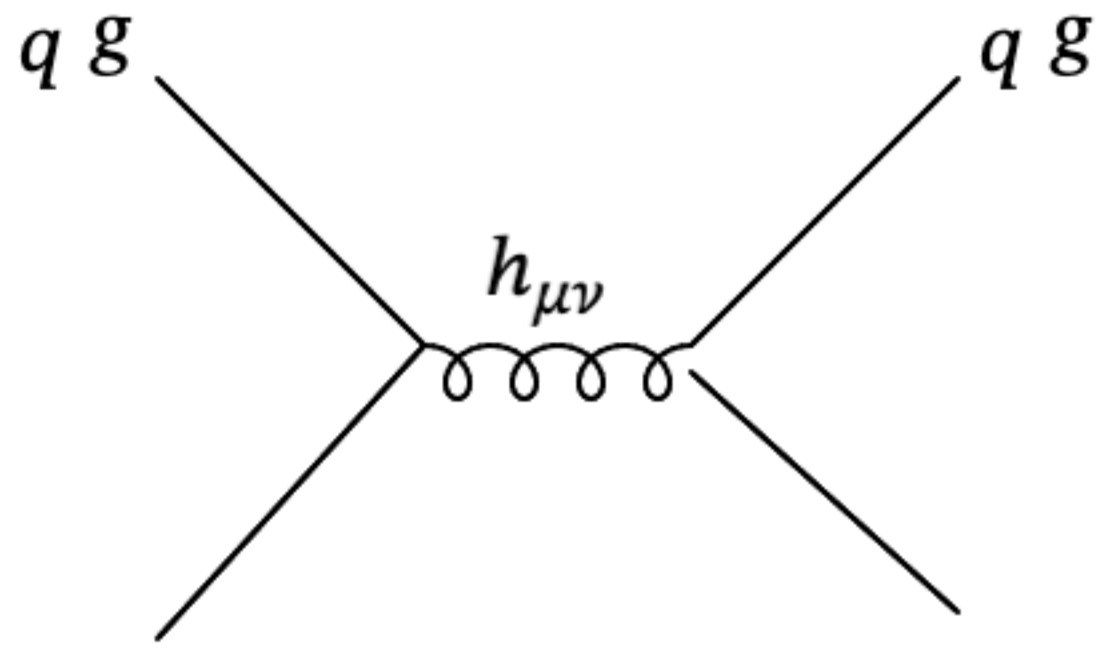
Spin-Independent direct detection cross section

$$\sigma^{\mathrm{SI}}(\Phi A\to\Phi A)\approx\frac{\mu_A^2}{144\pi m_A^2}\sum_{\Psi=\{u,d,s,c,b\}}\left(Zm_p^2\bar{\mathcal{M}}_\Psi^p+(A-Z)\,m_n^2\bar{\mathcal{M}}_\Psi^n\right)^2$$

Collider Bounds on KK Gravitons …



pp → $h_{\mu\nu}$ → jj



$$\mathcal{L} = -\frac{1}{\overline{M}_{Pl}} T^{\alpha\beta}(x) h^{(0)}_{\alpha\beta}(x) - \frac{1}{\Lambda_\pi} T^{\alpha\beta}(x) \sum_{n=1}^{\infty} h^{(n)}_{\alpha\beta}(x) \, .$$

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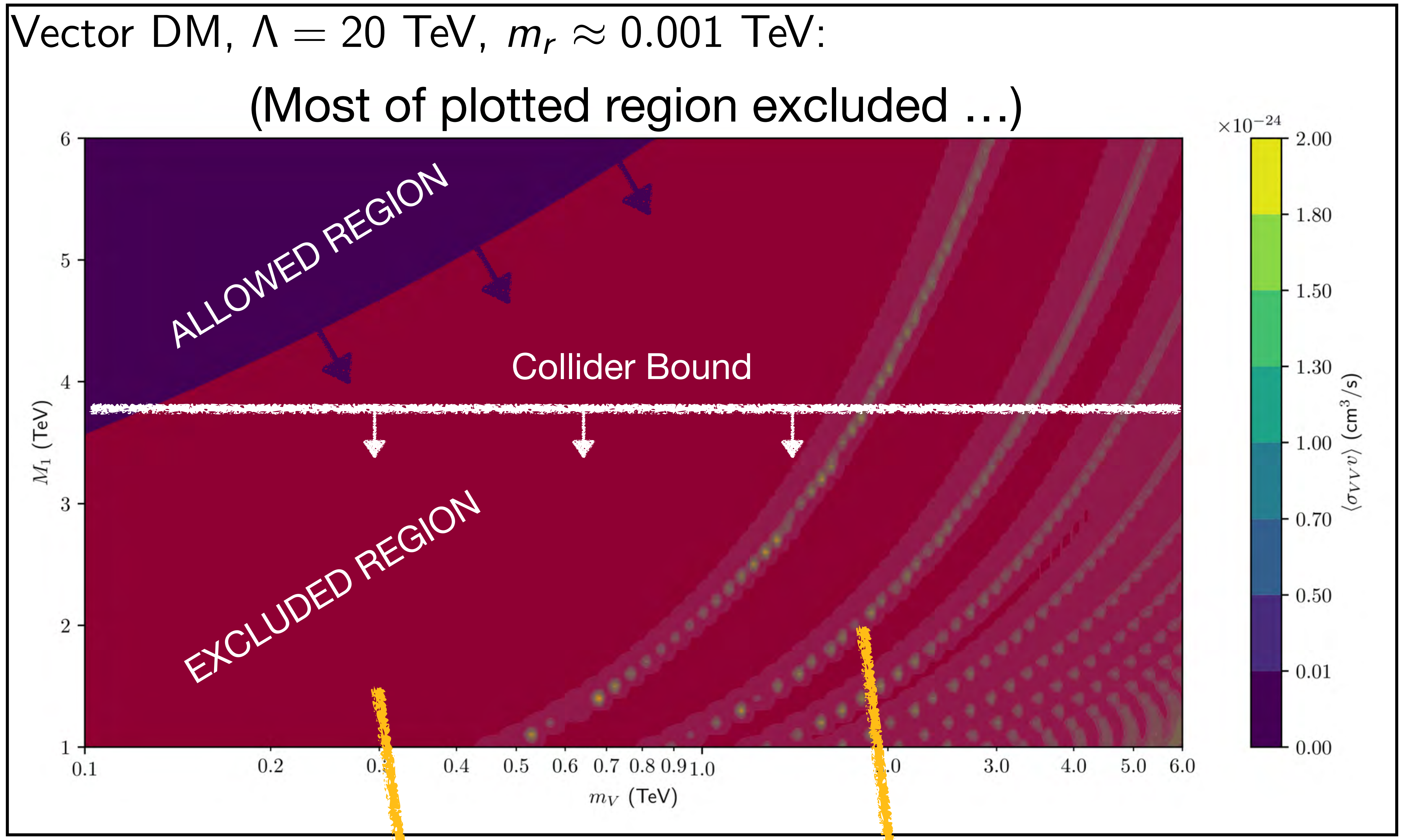
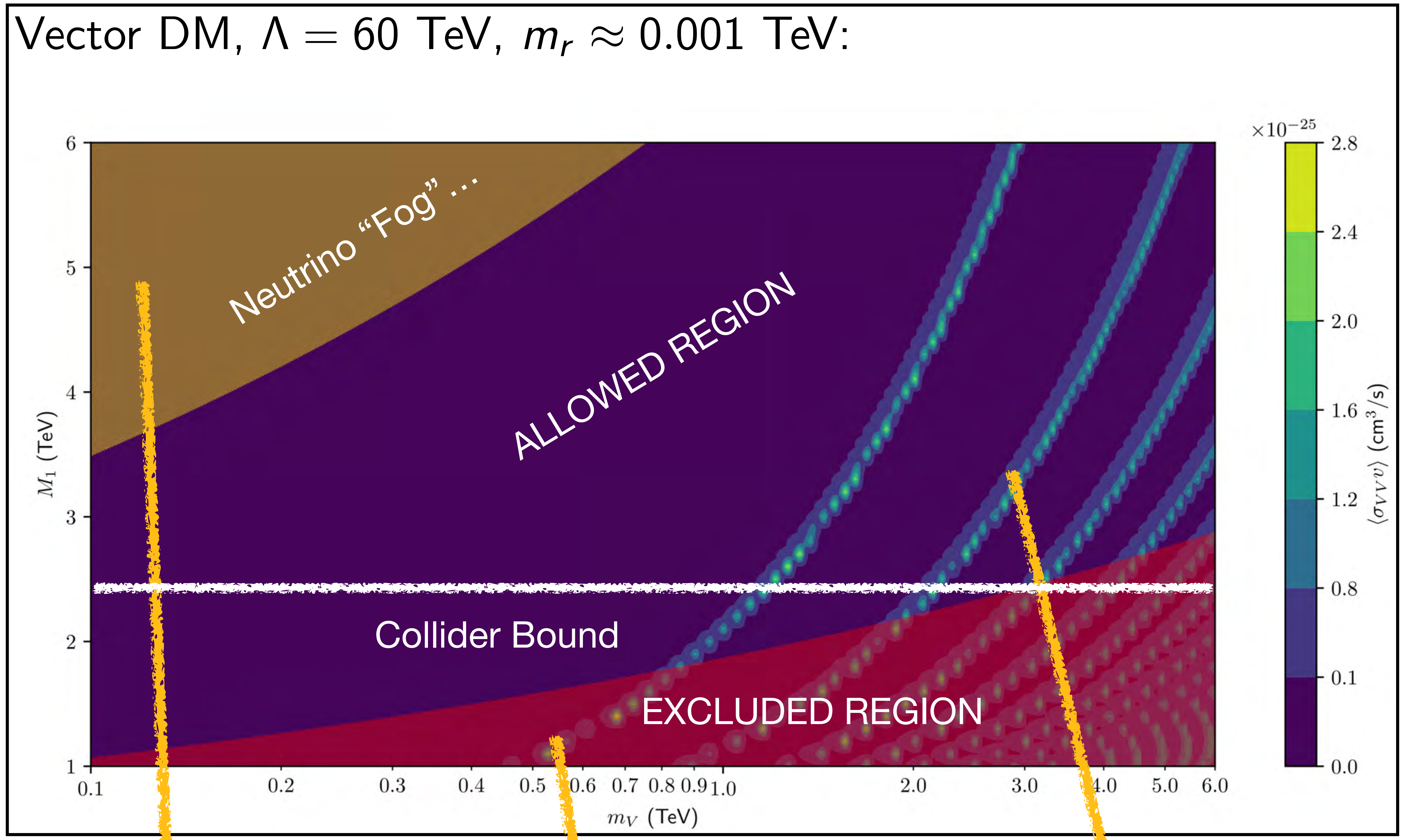
KK Portal Dark Matter Constraints

Relic density, direct detection, collider combination





Ruled out by LZ limits 2023

Relic density constraint is satisfied

Solar neutrino floor

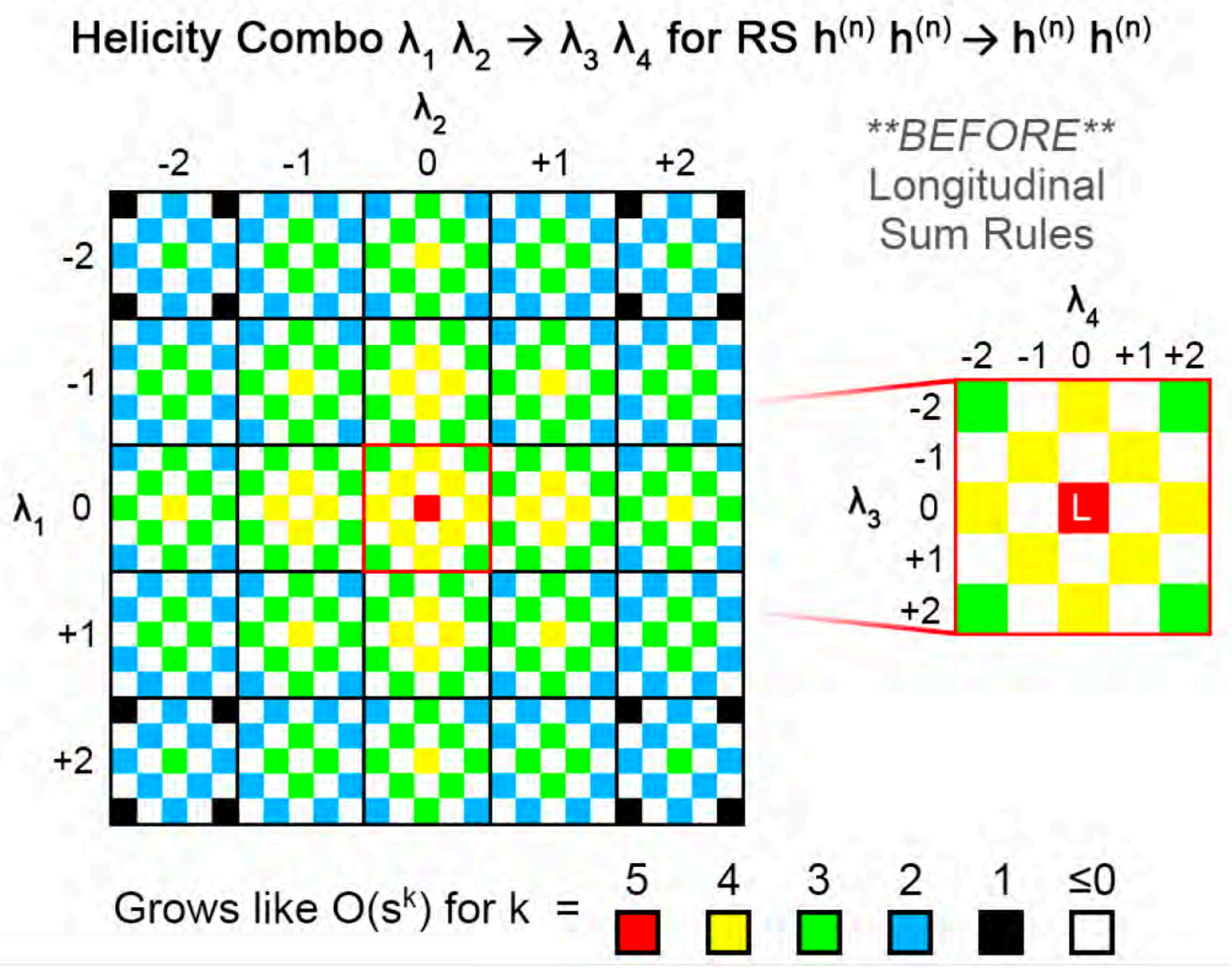
Ruled out by LZ limits 2023

Relic density constraint is satisfied

- Scalar DM never satisfy the relic density for sensible choices of parameters
- Fermion DM are ruled out by LZ direct detection limits
- Vector DM works in "portal region" where $m_G \cong 2m_{DM}$

Spin-2 KK Scattering Amplitudes

- Individual contributions to the scattering amplitudes grow as fast as $O(s^5)$ in unitary gauge.
- Cancellation occurs between the diagrams - enforced by relationships between couplings and KK masses (proven).
- These cancellations are the result of the underlying 5D diffeomorphism invariance of the theory.
- Alternatively: examine theory in `t-Hooft-Feynman gauge, including unphysical spin-0 and spin-1 "Goldstone" bosons.
- Equivalence theorem insures that helicity-0 spin-2 scattering amplitudes equal those of the corresponding Goldstone scalars, which are O(s) by power-counting.
- Same remains true in a model with a stabilized extra dimension, and a massive radion.



KK Graviton Portal Dark Matter

- Model determined (for fixed M_{Pl}) by type of DM particle, Λ_π, m_G, m_{DM}
  - All cross-sections scale like s/Λ^4_π
  - Scalar and Fermion DM ruled out
  - Vector DM allowed, Λ above 10 TeV range, in "portal region" $m_G \cong 2m_{DM}$
  - Allowed region can reach to the neutrino floor, and below …